Efficient Coding

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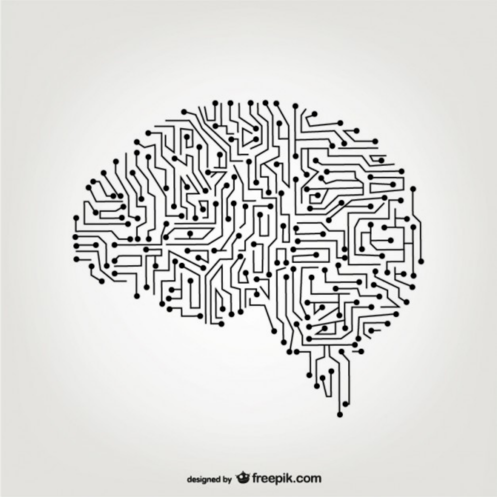
2021

Levels of modeling

- Descriptive (what)
- Mechanistic (how)
- Interpretive (why)



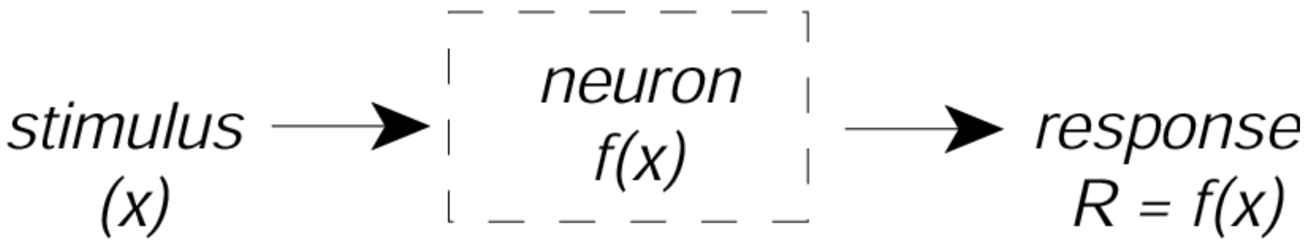
Levels of modeling

- Fitting a receptive field model to experimental data (e.g., using spike-triggered stimuli; you've seen)

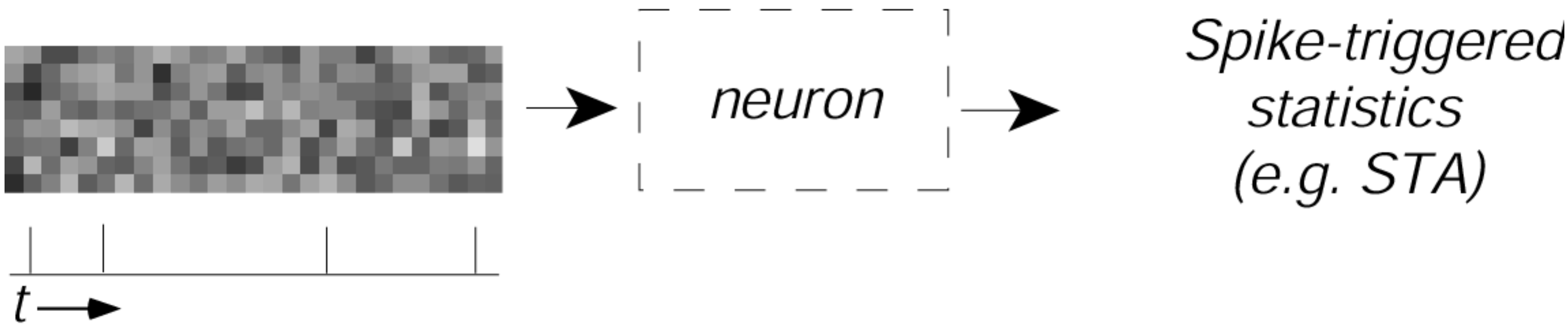
Versus

- Deriving receptive field model based on theoretical principles (e.g., statistical structure of scenes)

Fitting a model to data



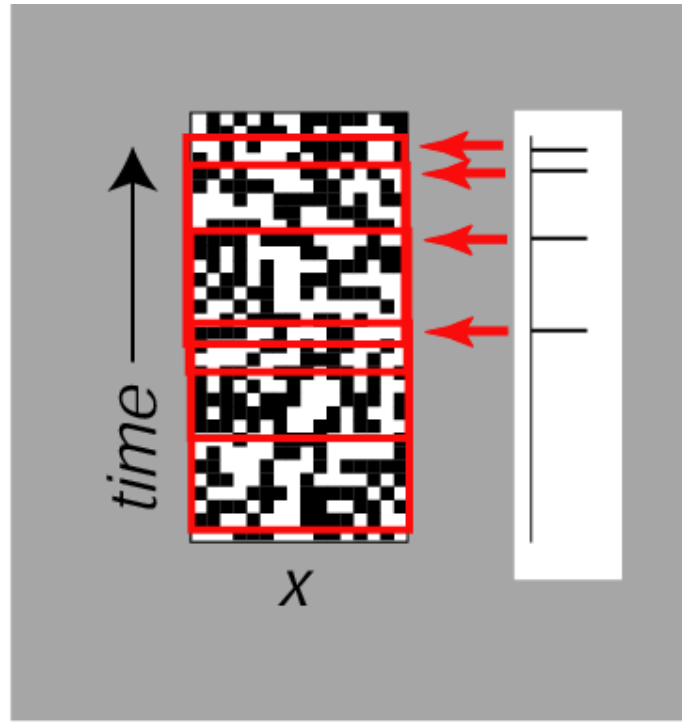
Fitting a model to data



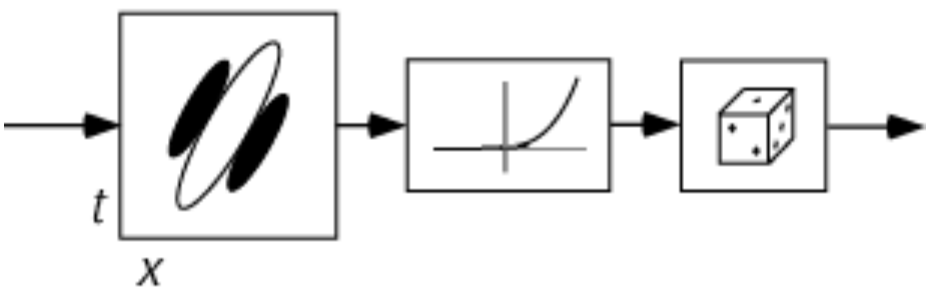
Primary visual cortex

STA

Average of
spike-triggered
stimuli

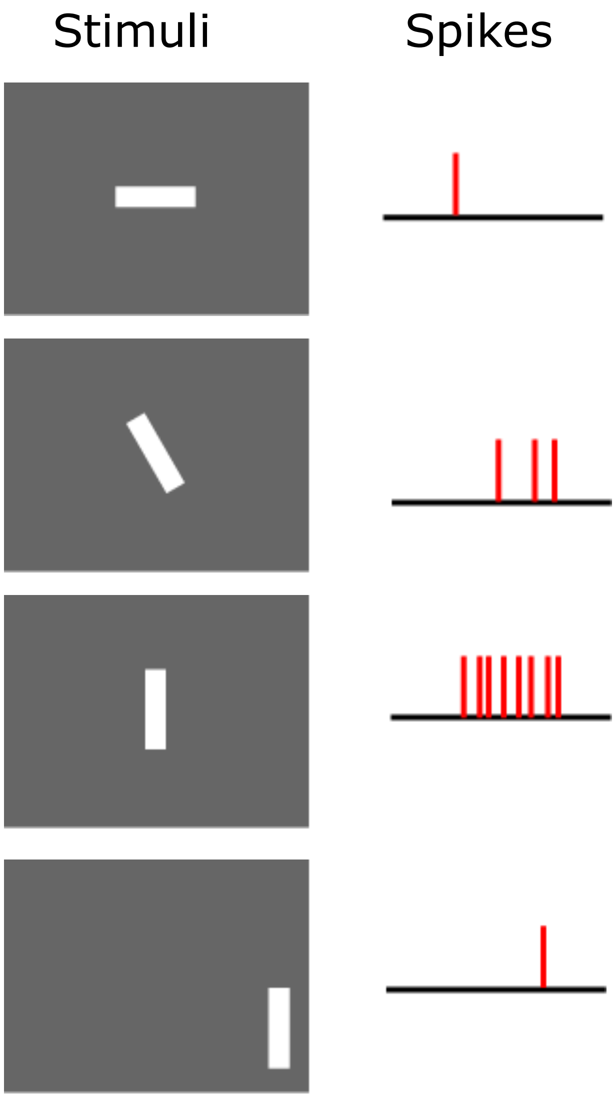


Model:



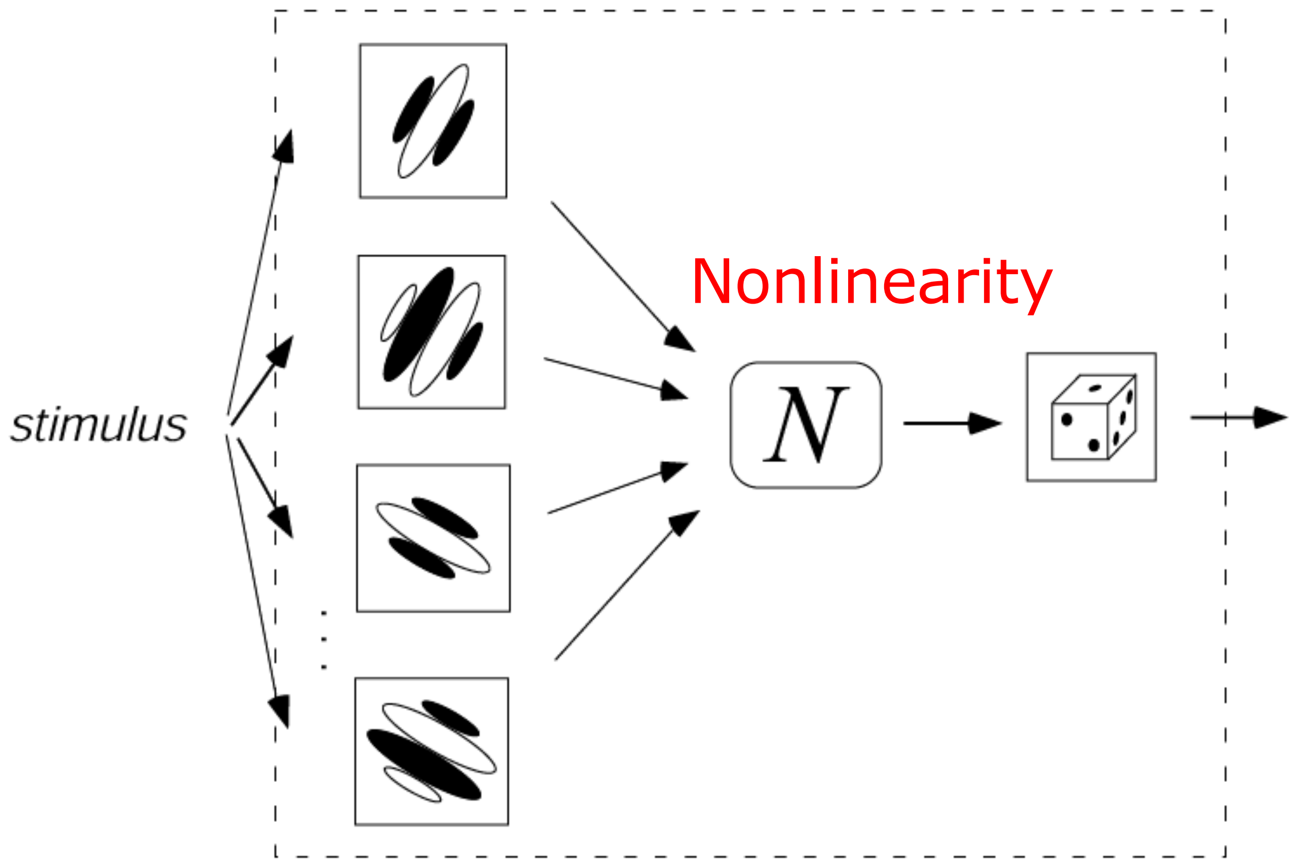
Primary visual cortex

Hubel and Wiesel, 1959



Receptive field (filter)

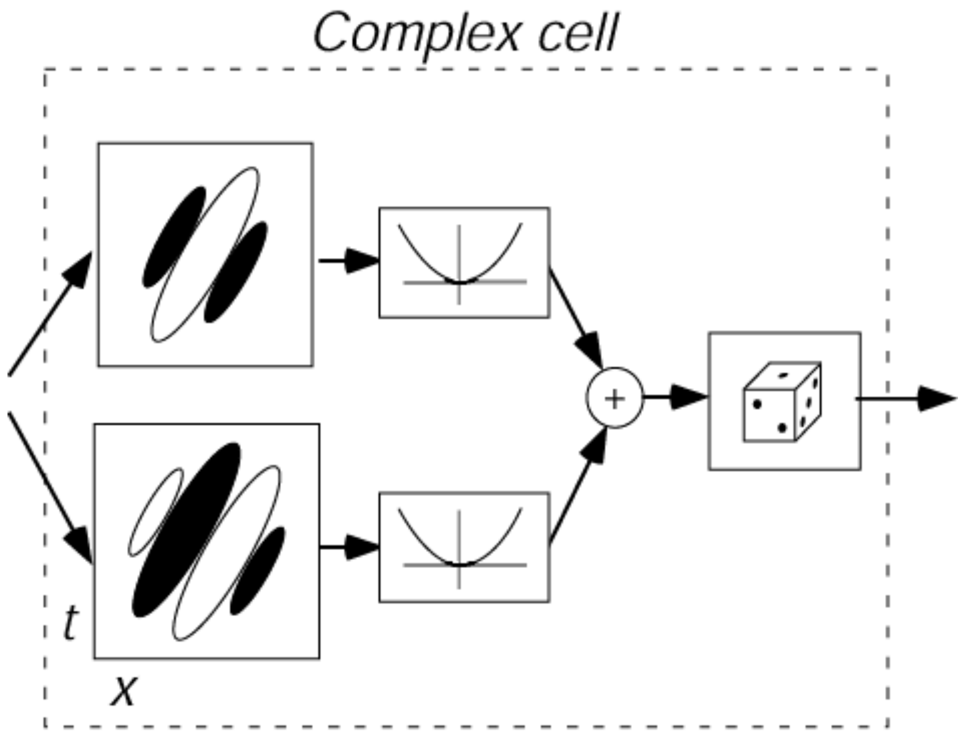
Fitting a model to spike data



Fitting a model to spike data

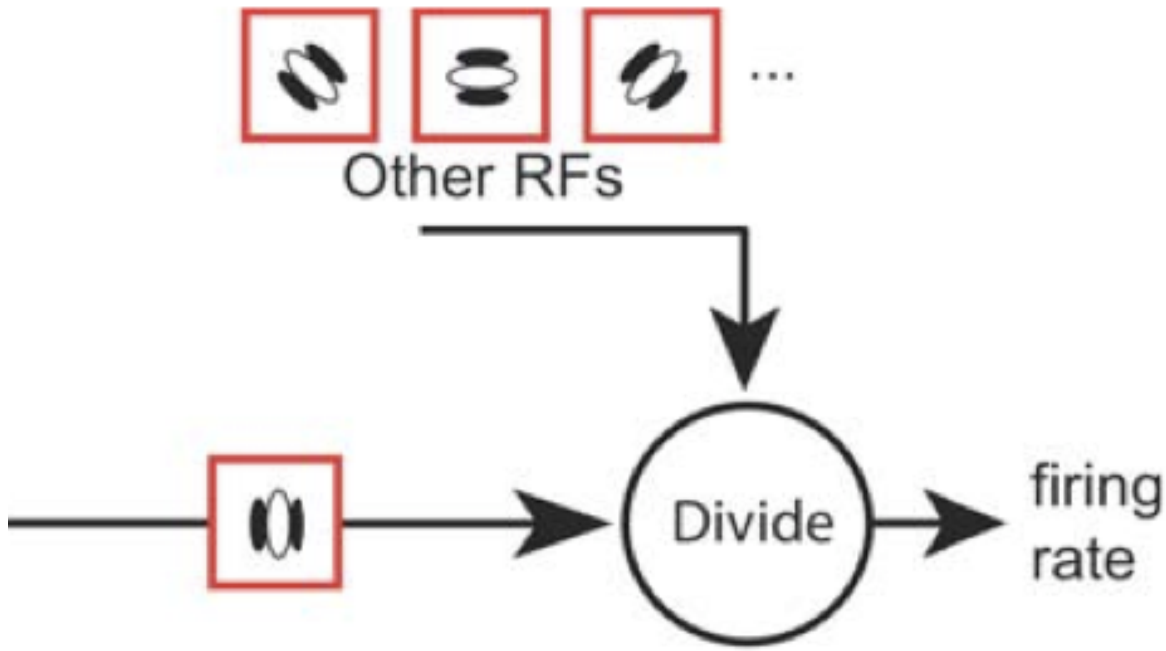
What kind of nonlinearities?

Complex cell

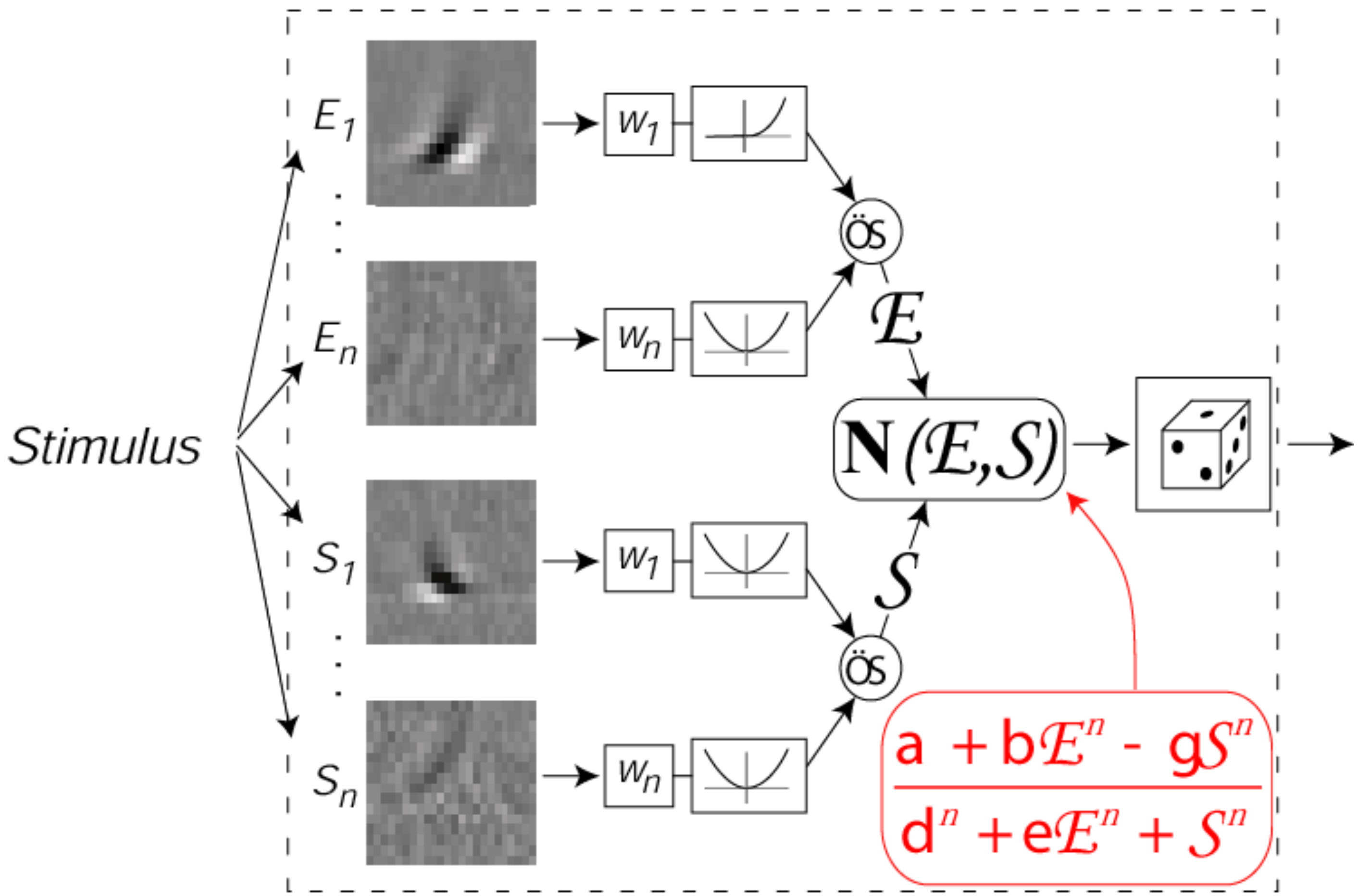


''

Divisive normalization



Fitting a model to spike data



Rust et al. 2005; Schwartz et al. 2006

Levels of modeling

- Fitting a receptive field model to experimental data (e.g., using spike-triggered stimuli)

Versus

- Deriving receptive field model based on theoretical principles (e.g., statistical structure of scenes)—adding an interpretive layer.

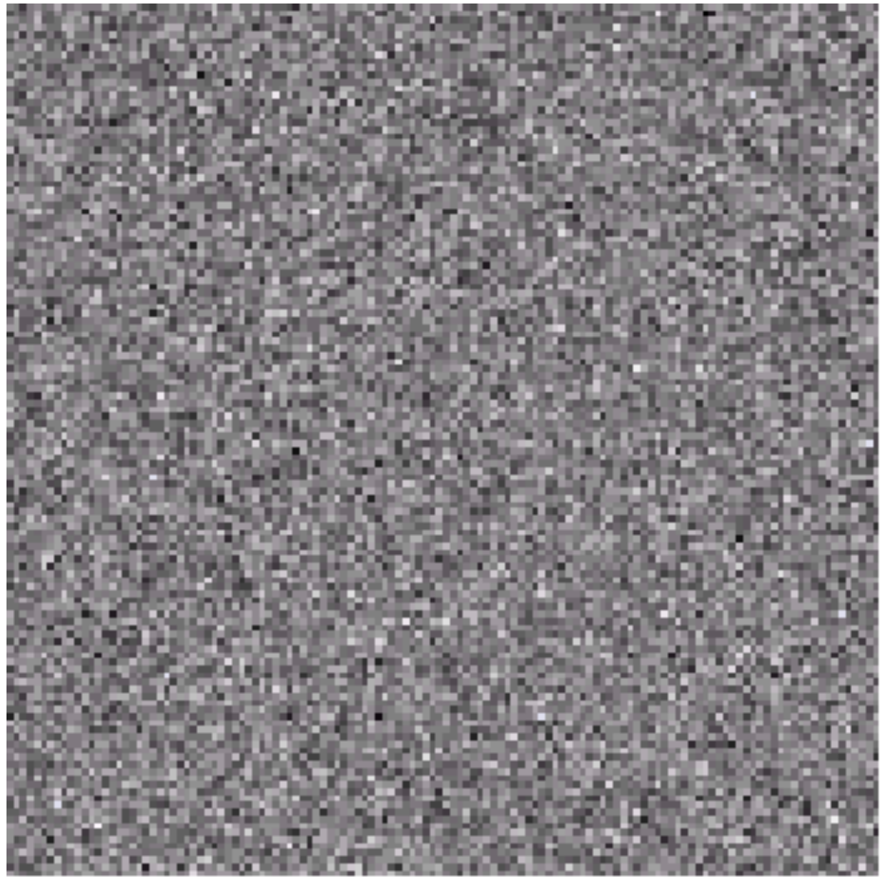
Complementary question

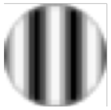
Can we derive or constrain a neural model by understanding statistical regularities in scenes?

Complementary question

Appealing hypothesis: brain evolved to capture probabilistic aspects of the natural environment

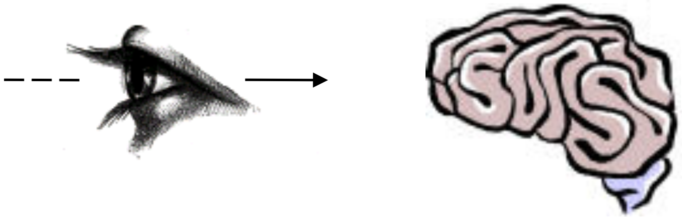
Unlike…



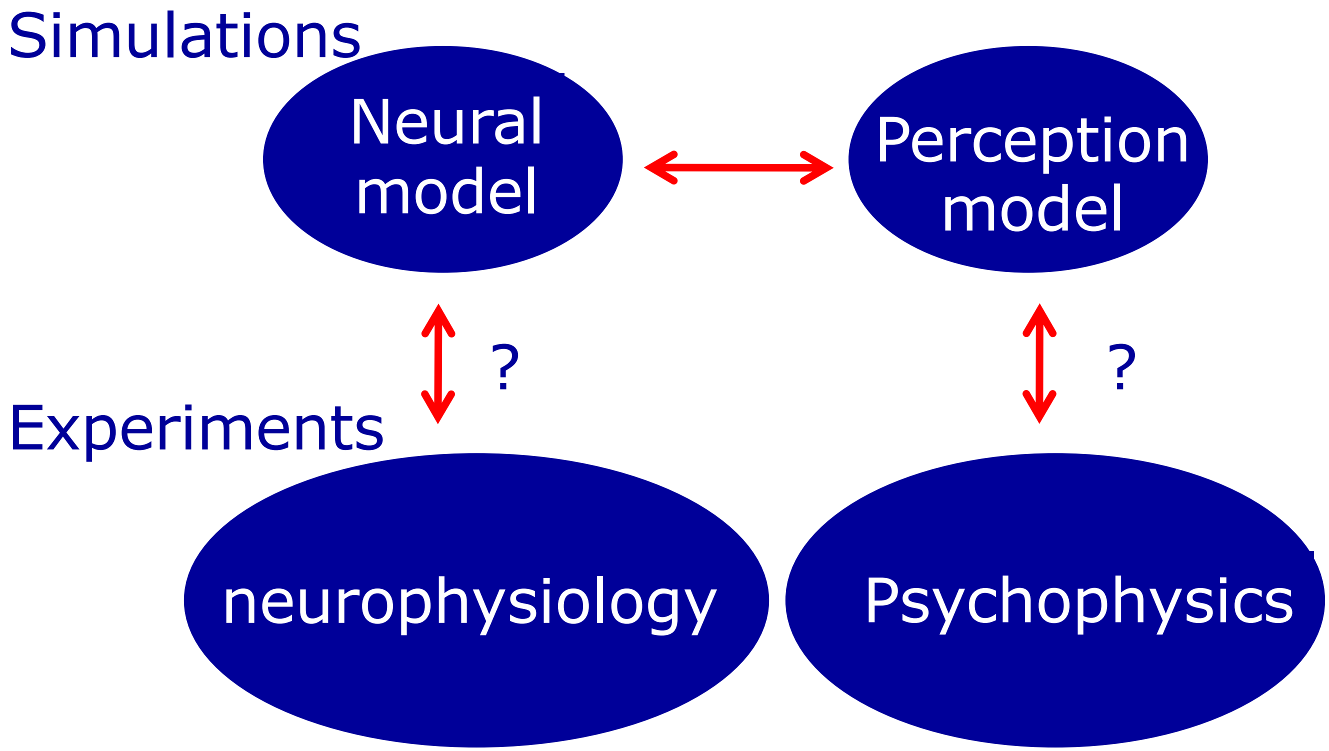


Also quite different from traditional experimental stimulus in vision experiments…

Visual representation

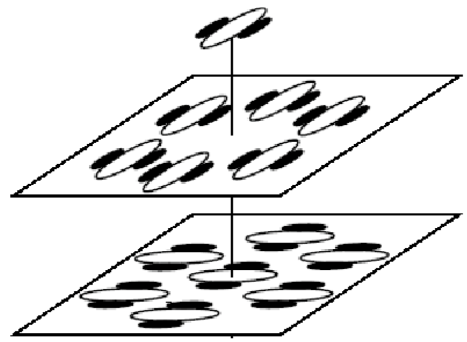


Visual input



- Goals: Principled and predictive understanding

Building model from scene stats



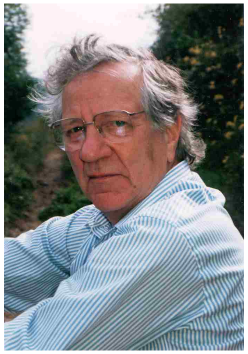
- Cortical computations as interactions of RFs across space, orientation, etc.
- RFs and interaction constrained by scene statistics? Can we derive them?

Theory

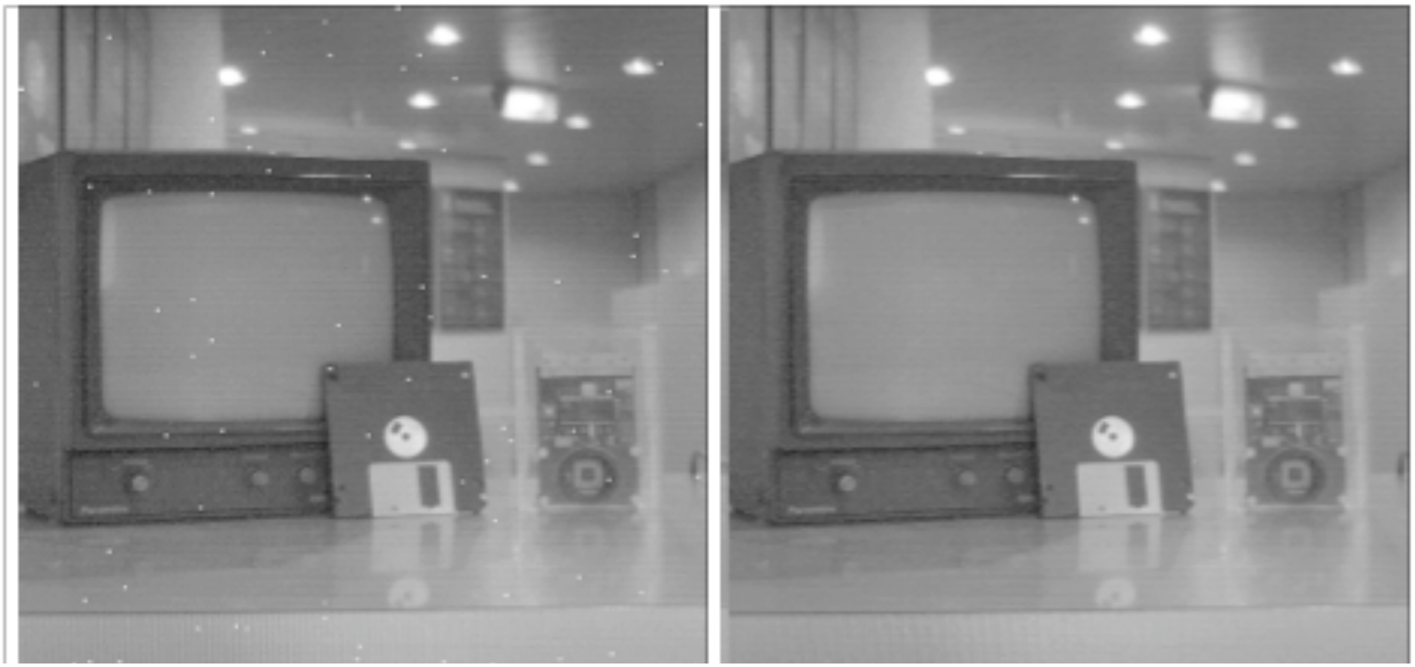
- **Locke:** The mind is a “tabula rasa” and only filled with knowledge after sense experience

- **Helmholtz:** perception as inference of the properties of sensory stimuli

- **Attneave, Barlow:** Hypothesized in the 1950s that sensory processing matched to statistics of environment (reduce redundancy; increase independence)



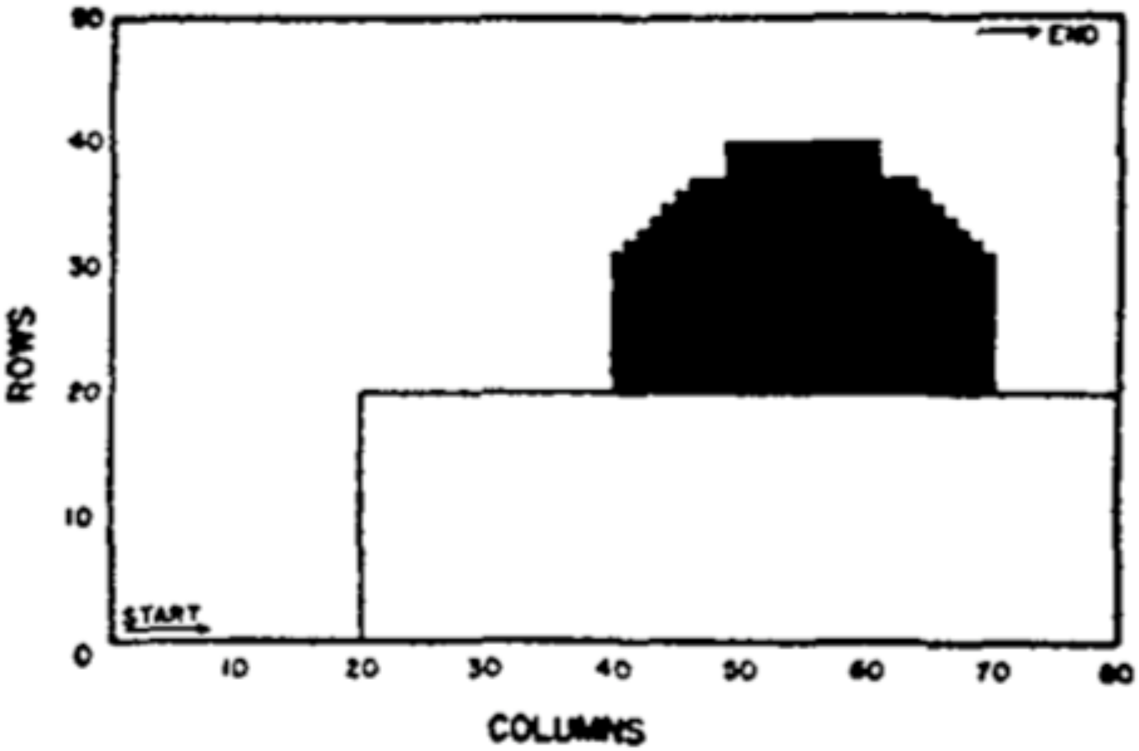
Images are spatially redundant



Kersten, 1992 (psychophysics);

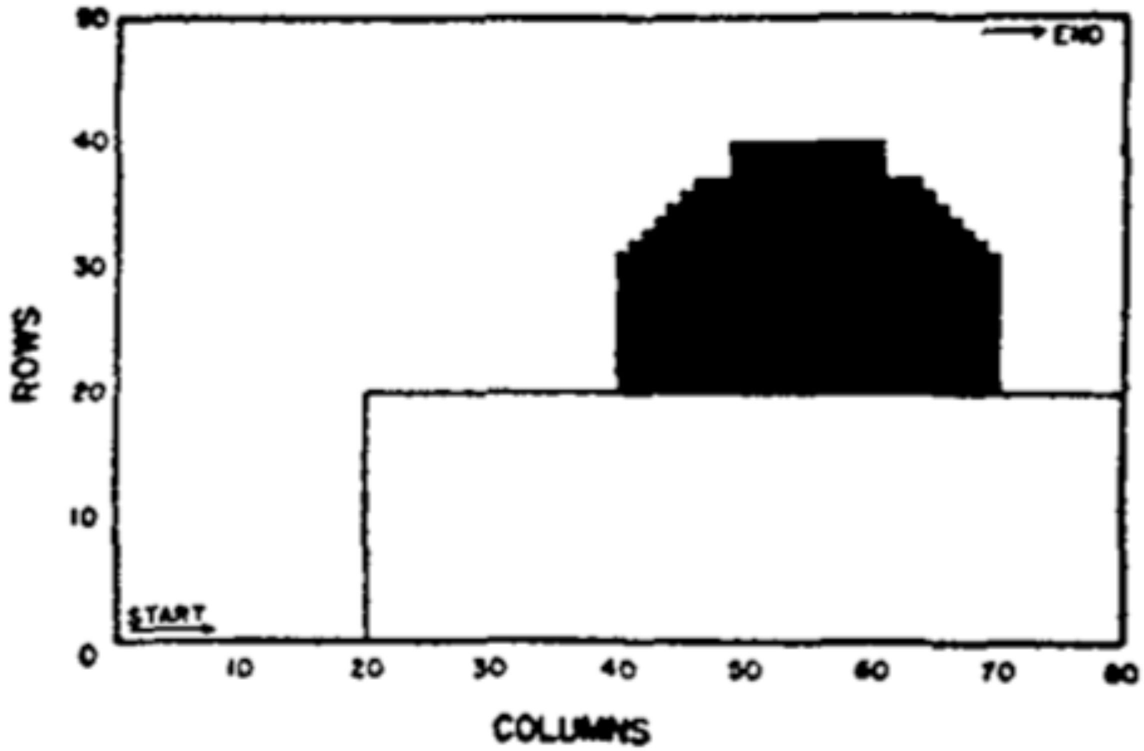
Dierickx and Meynants, 1987 (computer)

Images are spatially redundant



Attneave 1951; “guessing game”

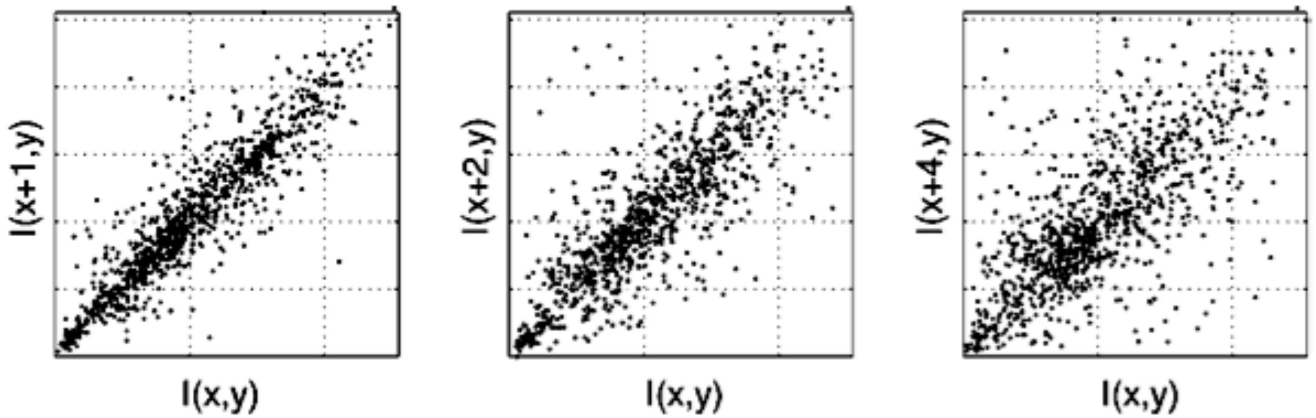
Images are spatially redundant

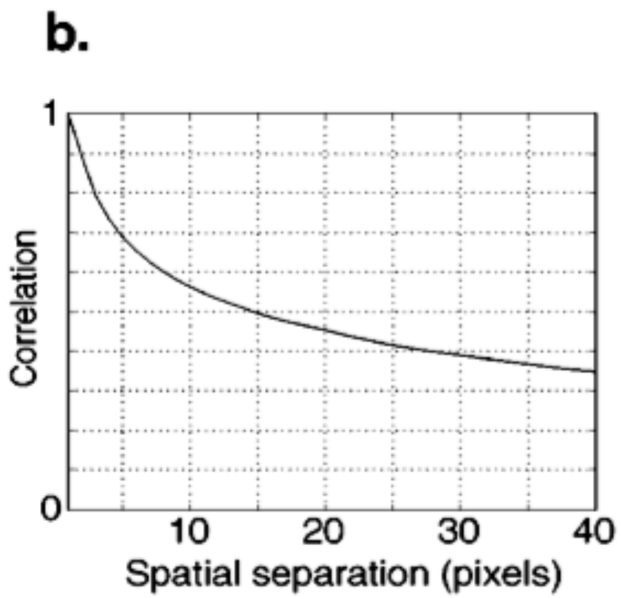


 Attneave 1951; “ink bottle on the corner of the desk”

Images are spatially redundant

Statistics of images show dependencies





Simoncelli and Olshausen review, 2001

Scene statistics approaches

Two main approaches for studying scene statistics

1. Bottom-up

2. Top-down, generative

Scene statistics approaches

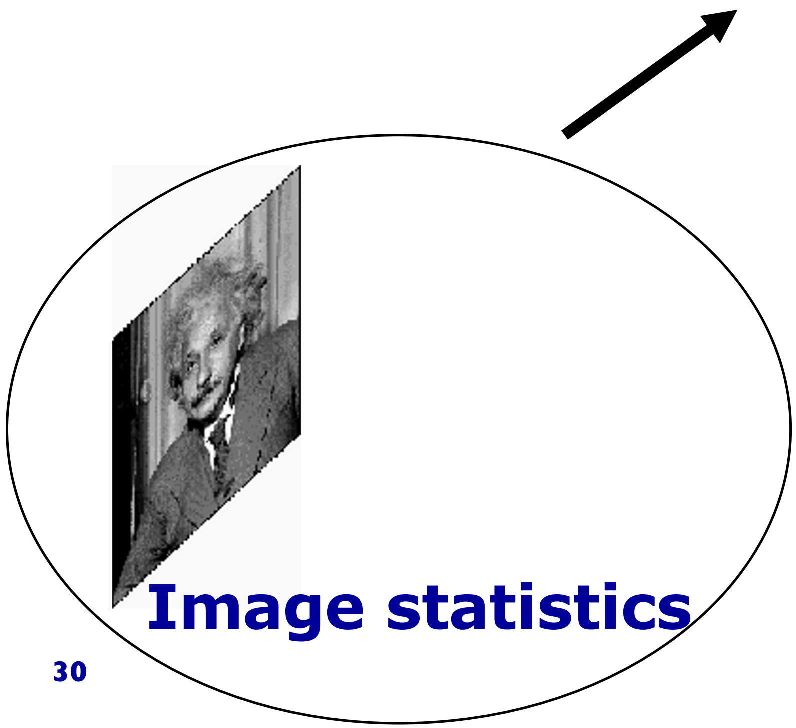
Two main approaches for studying scene statistics

1. Bottom-up (This class!)

2. Top-down, generative

Bottom-up approach

Choose and manipulate projections,
to optimize probabilistic and
information-theoretic metrics



Bottom-up approach

We'll take a small detour and talk about Information Theory…

Image statistics

Intro Information Theory

Redundancy:

- Marginal distribution (eg., in English “a” more often than “q”)

- Joint distribution (eg, ”sh” more often than “sd”)

- Analogous to images marginal and joint… (later)

Intro Information Theory

Redundancy and relation to coding in bits:

BABABABADABACAABAACABDAAAAABAAAAAAAADBCA

$A \rightarrow 00$

$B \rightarrow 01$

$C \rightarrow 10$

$D \rightarrow 11$

0100010001000100110001001000000100001000
0111000000000001000000000000000011011000

$A \rightarrow 0$

$B \rightarrow 10$

$C \rightarrow 110$

$D \rightarrow 111$

1001001001001110100110001000110010111000
001000000000111101100

Hyvarinen et al. book, 2009

Intro Information Theory

Entropy:

$$H(y) = -\sum_{y} p(y) \log_2 p(y)$$

- Measure of uncertainty or how interesting
- Always positive and equal to zero iff outcome is certain
- Log base 2 – expressed in bits
- Relates to minimal coding length

Intro Information Theory

Entropy:

$$H(y) = -\sum_{y} p(y)\log_2 p(y)$$

- Example: P(A)=1/2; P(B)=1/4; P(C)=1/8; P(D)=1/8
- Entropy = 1.75 bits (compared to 2 bits if all equal)

Intro Information Theory

Entropy:

$$H(y) = -\sum_{y} p(y)\log_2 p(y)$$

- If there are 2 possible outcomes with probability p and 1-p, **when is the entropy maximal**?

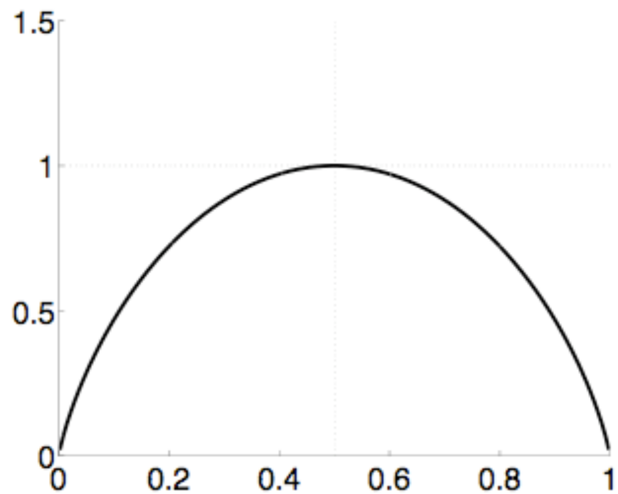
Intro Information Theory

Entropy:

$$H(y) = -\sum_{y} p(y)\log_2 p(y)$$

Example: possible outcomes: a, b

entropy



P(y=a)

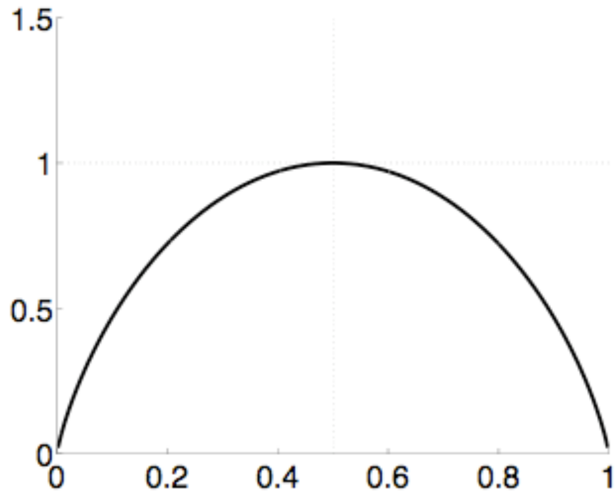
Intro Information Theory

Entropy:

$$H(y) = -\sum_{y} p(y)\log_2 p(y)$$

Example: possible outcomes: a, b

entropy



P(y=a)

Maximum entropy when most random (0.5), or more generally for uniform distribution

Fig. 8.
probab

Intro Information Theory

We've thus far looked at marginal distributions through one channel; we would like to also look at joint…

Conditional Entropy:

$$H(y \mid x) = -\sum_x p(x) \sum_y p(y \mid x) \log_2 p(y \mid x)$$

How much entropy left in y when we know x

Intro Information Theory

We've thus far looked at marginal distributions through one channel; we would like to also look at joint…

Conditional Entropy:

$$H(y \mid x) = -\sum_{x} p(x) \sum_{y} p(y \mid x) \log_2 p(y \mid x)$$

averaged over all x	How much entropy left in y when we know x

Intro Information Theory

We’ve thus far looked at marginal distributions through one channel; we would like to also look at joint…

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averaged over all x	How much entropy left in y when we know x

- What happens when x and y are independent? Dependent? Equal?

Intro Information Theory

Conditional Entropy:

$$H(y \mid x) = -\sum_{x} p(x) \sum_{y} p(y \mid x) \log_2 p(y \mid x)$$

- How much entropy left in y when we know x, averaged over all x

- What happens when x and y are independent? Dependent?

Independent: $H(y \mid x) = H(y)$

Dependent: $H(y \mid x) < H(y)$

Equal: $H(y \mid x) = 0$

Intro Information Theory

Mutual information:

$$I(x,y) = H(y) - H(y \mid x)$$

- What is the mutual information if x and y are independent?

Intro Information Theory

Mutual information:

$$I(x,y) = h(y) - h(y \mid x) = \ldots$$

$$\sum_{x,y} p(x,y) \log_2 \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

- What is the mutual information if x and y are independent?
- Also Kullback-Leibler between…

Intro Information Theory

Marginal and joint entropy:

$$H(y_1, y_2, \ldots y_n) \leq H(y_1) + H(y_2) + \ldots + H(y_n)$$

Equality iff independent: $p(y_1, y_2, \ldots y_n) = p(y_1)\,p(y_2) \ldots p(y_n)$

Intro Information Theory

Marginal and joint entropy:

$$H(y_1, y_2, \ldots y_n) \leq H(y_1) + H(y_2) + \ldots + H(y_n)$$

Equality iff independent: $p(y_1, y_2, \ldots y_n) = p(y_1)p(y_2)\ldots p(y_n)$

Maximal entropy when:

- Outputs through a single channel as random as possible (but subject to constraints on channel)
- Independent. In general, hard to achieve. Restrict to, eg, linear.

Intro Information Theory

Redundancy; we can optimize…

- Marginal distribution (eg., in English “a” more often than “q”)
- Joint distribution (eg, ”sh” more often than “sd”)

What about images…?

Bottom-up approach

Choose and manipulate projections,
to optimize probabilistic and
information-theoretic metrics

We'll go through past examples in the field, building up to more recent approaches…

Image statistics

Bottom-up approach

Choose and manipulate projections,
to optimize probabilistic and
information-theoretic metrics

Image statistics

Optimizing marginal statistics

Efficient coding: single neuron fly vision

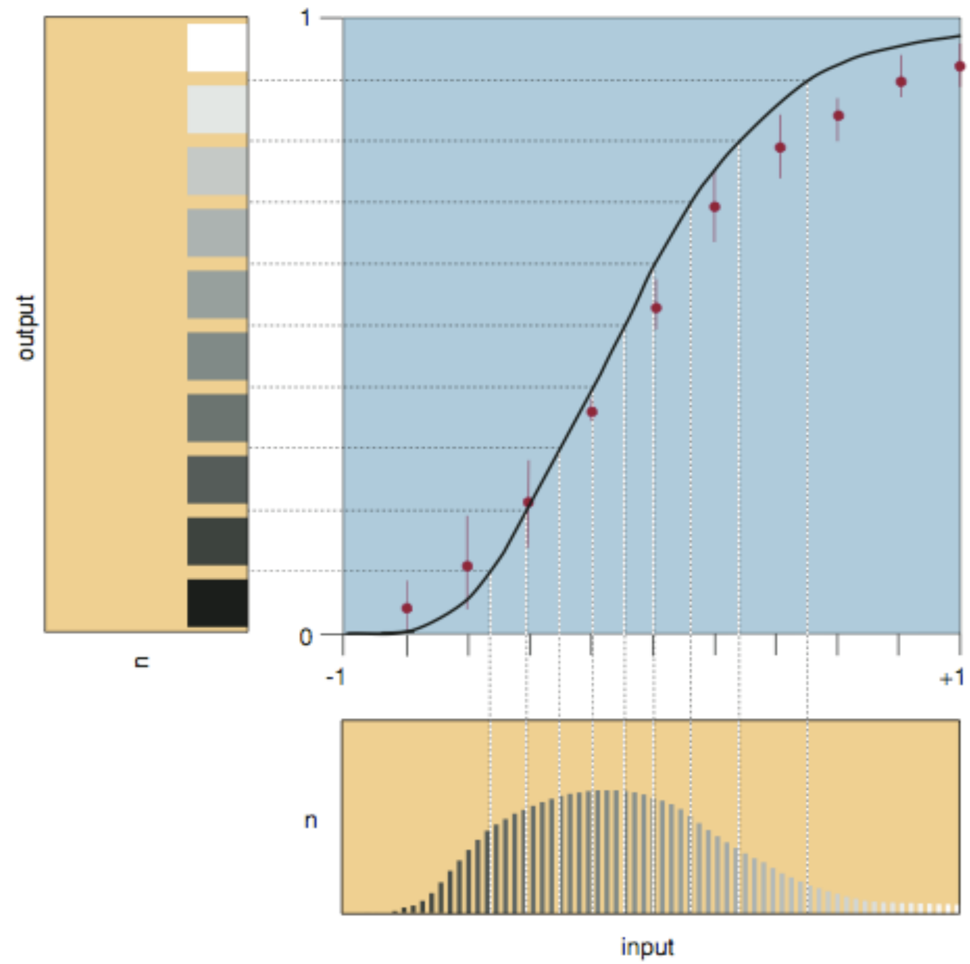
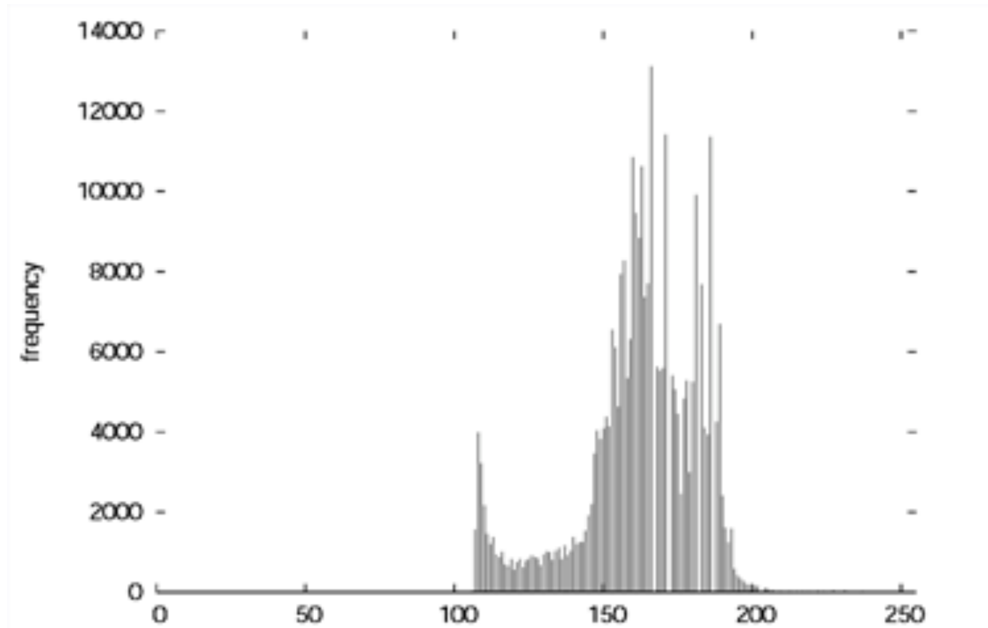


Figure from Olshausen & Field 2000; adapted from Laughlin 1981; Measured contrasts in natural scenes and showed that the membrane potential of fly visual neurons approximately transforms to uniform

Similar idea in image processing… Histogram equalization



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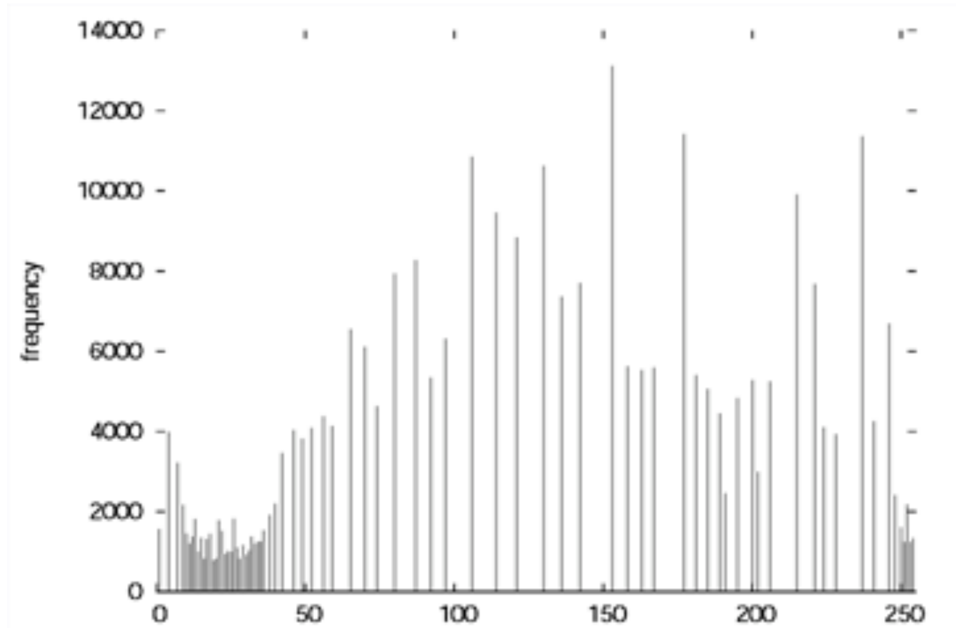

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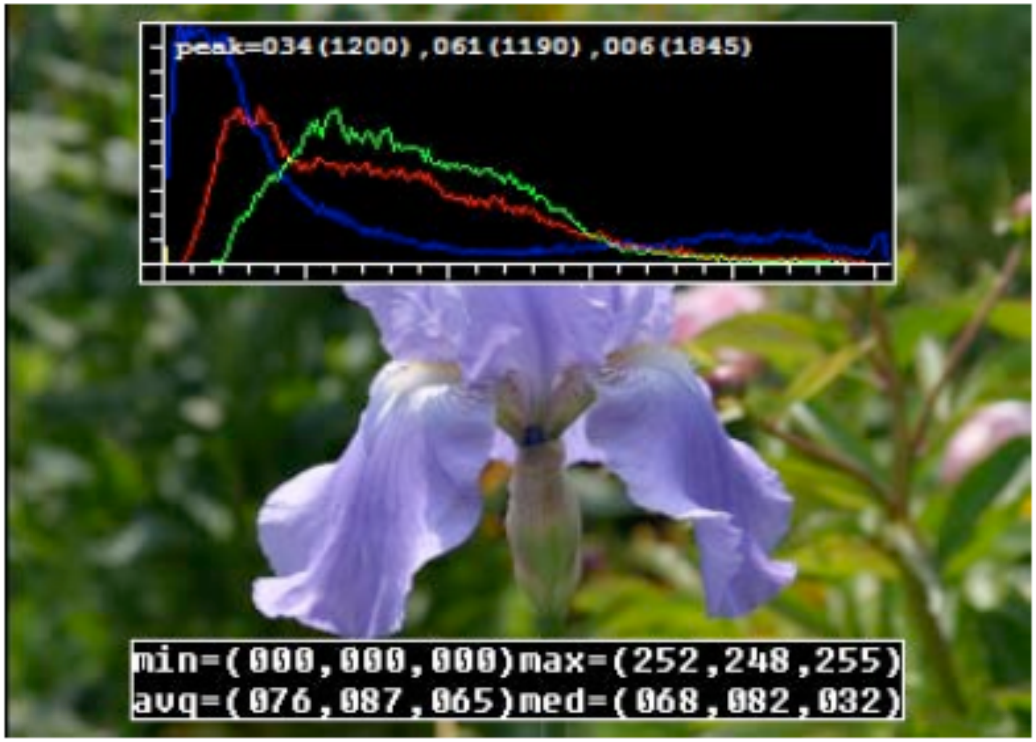
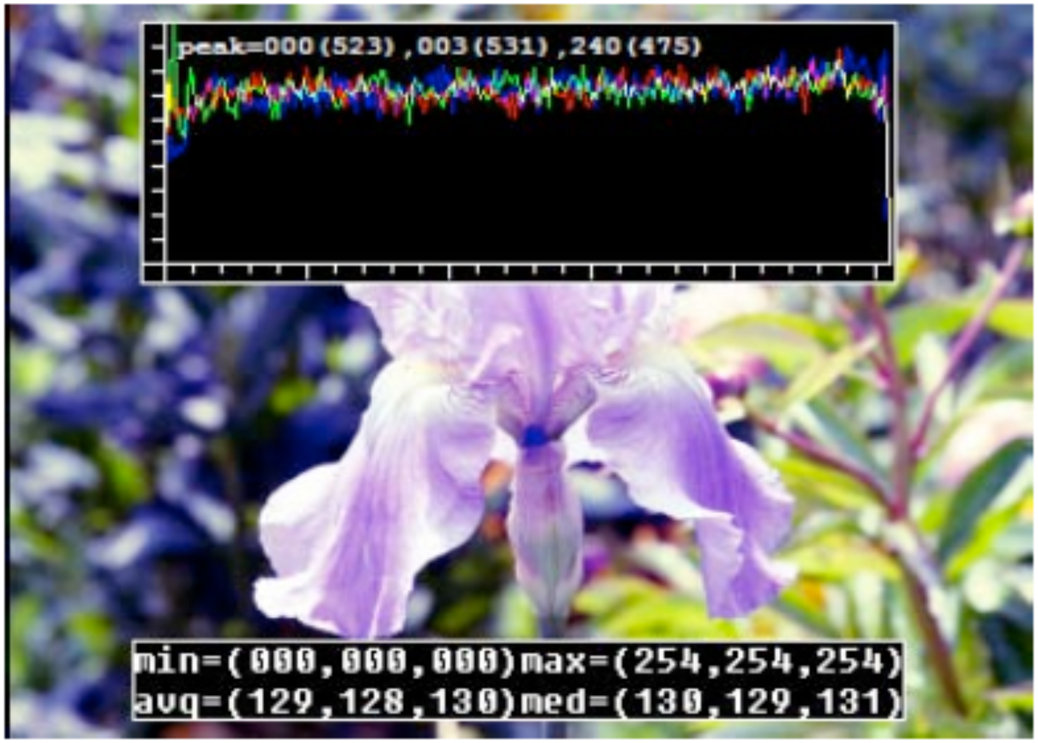


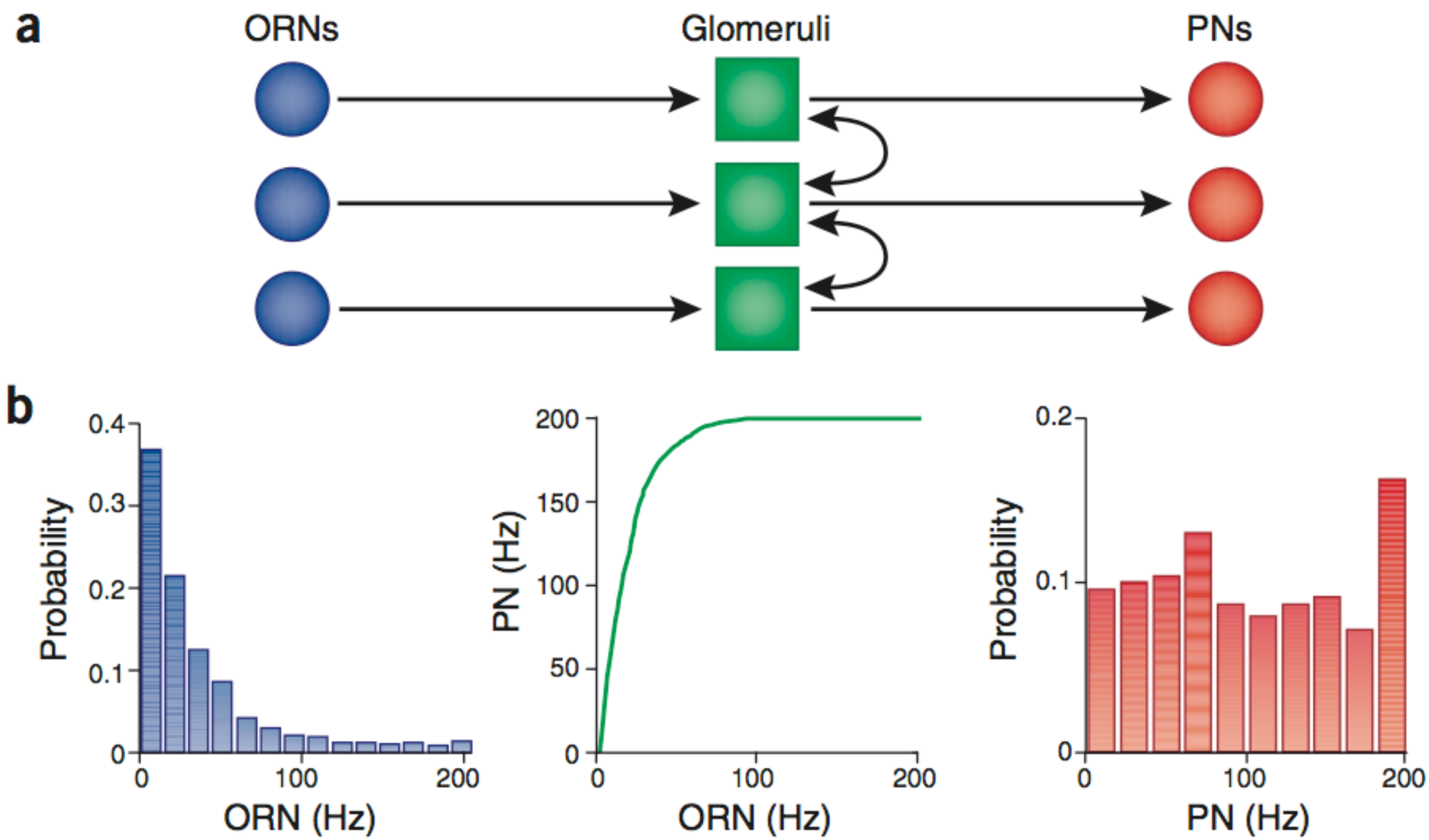
Richard Szeliski, Computer Vision Book 2010

Similar idea in image processing… Histogram equalization



Richard Szeliski, Computer Vision Book 2010

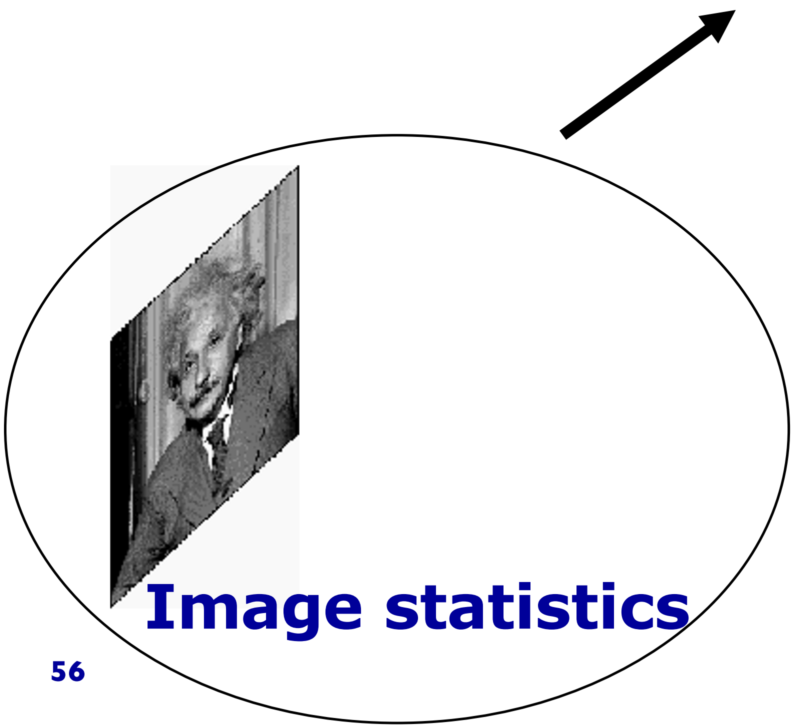
Efficient coding: fly olfaction



Abbott and Luo News and Views 2007;
After Bhandawat et al. 2007

Bottom-up approach

Choose and manipulate projections, to optimize probabilistic and information-theoretic metrics



Assuming a **linear** system and optimizing joint statistics: **decorrelation**

Bottom-up approach

Choose and manipulate projections,
to optimize probabilistic and
information-theoretic metrics

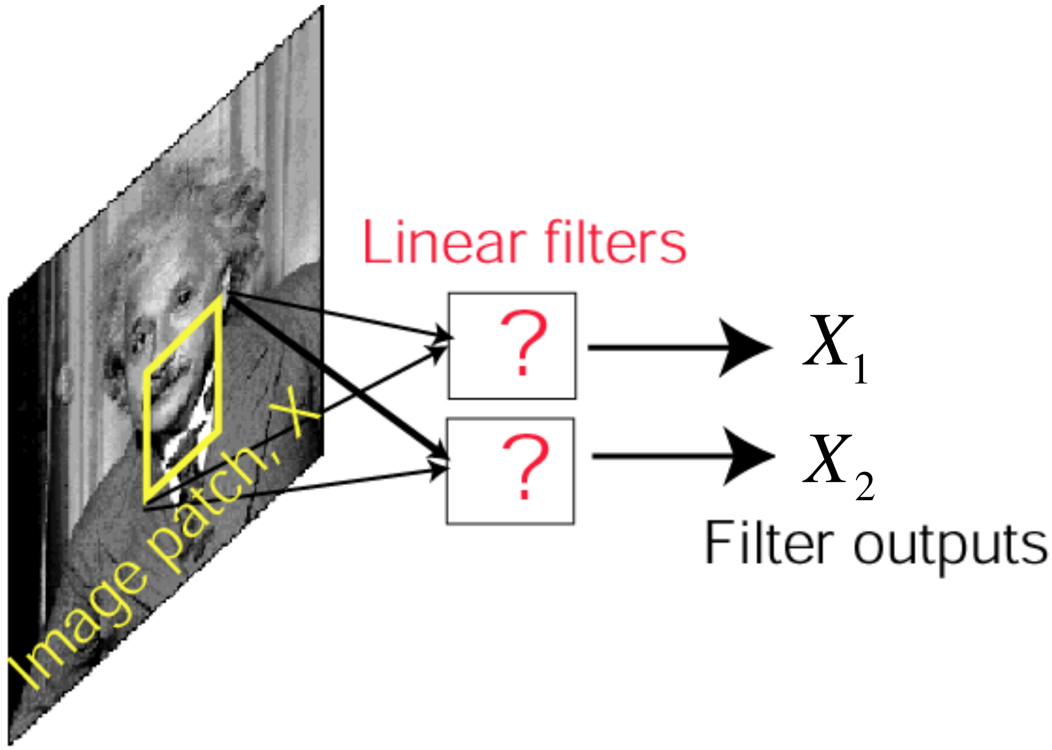
Image statistics

decorrelation

$$E[y_i y_j] = 0; i \neq j$$

Does this guarantee independence?

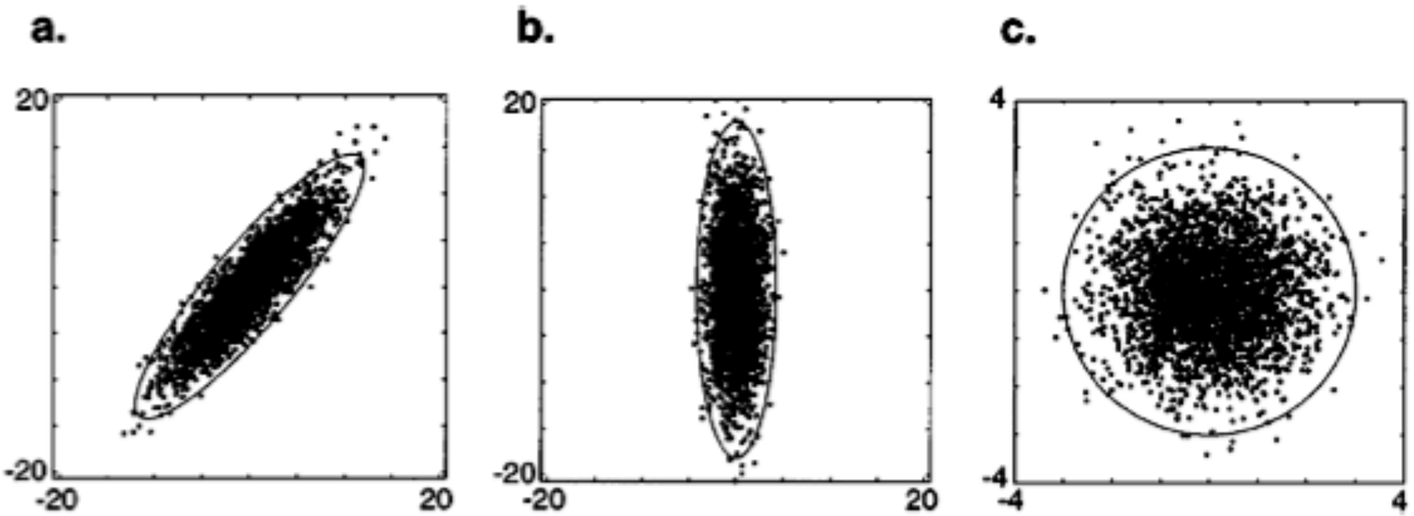
Linear Model: Theory



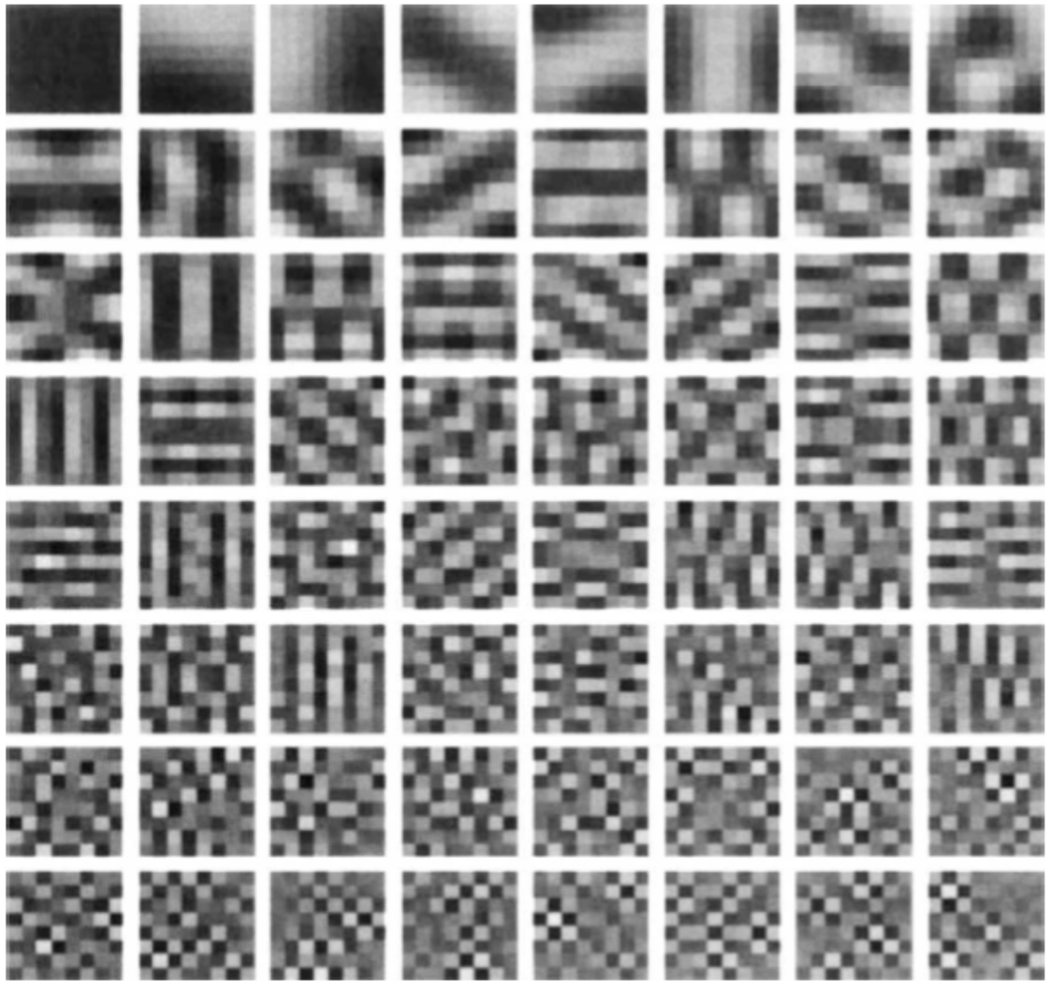
Find linear filters that decorrelate filter outputs to natural images

Geometric view (PCA)

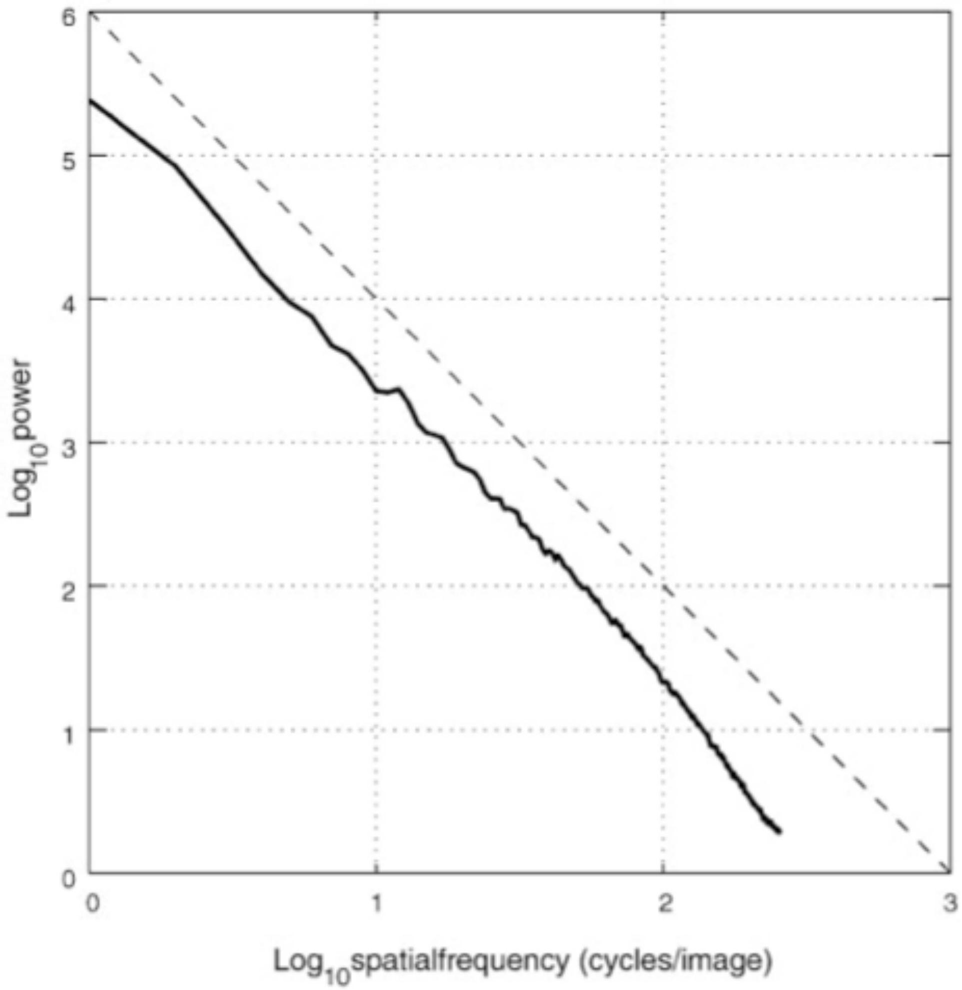
Gaussian distribution



Simoncelli & Olshausen review, 2001

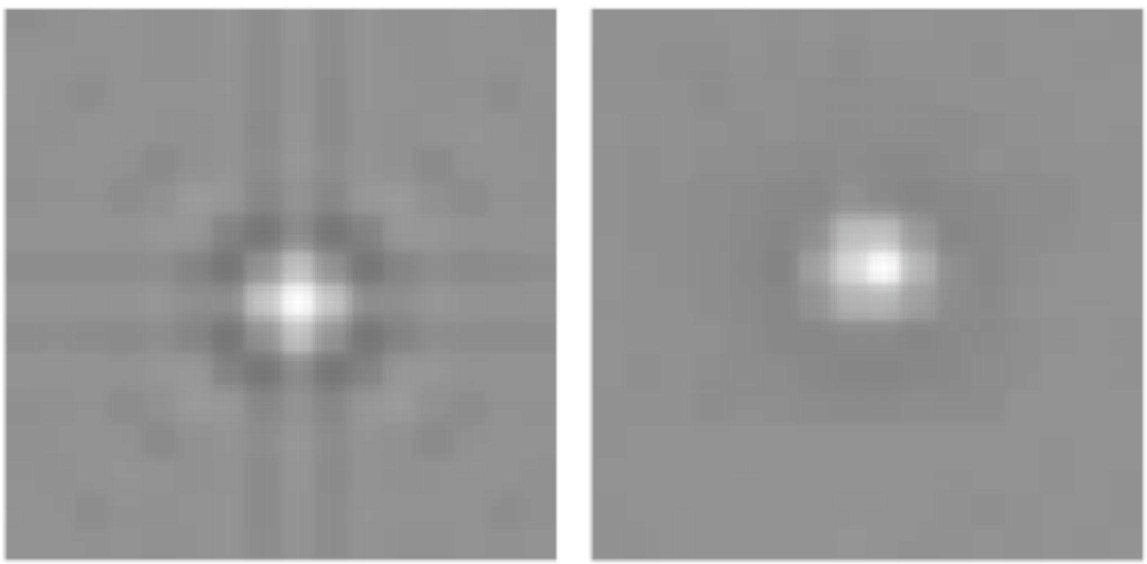


- Principal Component Analysis on image patches



• Power spectrum of natural images (from Simoncelli & Olshausen review)

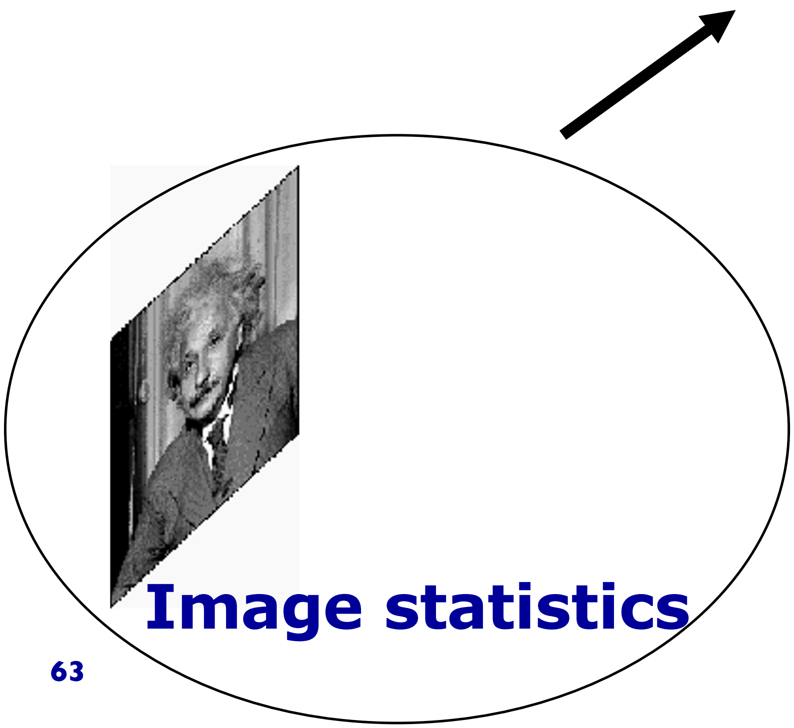
Bottom-up approach



- PCA and imposing extra constraints such as Spatially localized filters (from Hyvarinen book; see Atick & Redlich 1992; Zhaoping 2006)
- Remember decorrelated does not mean independent

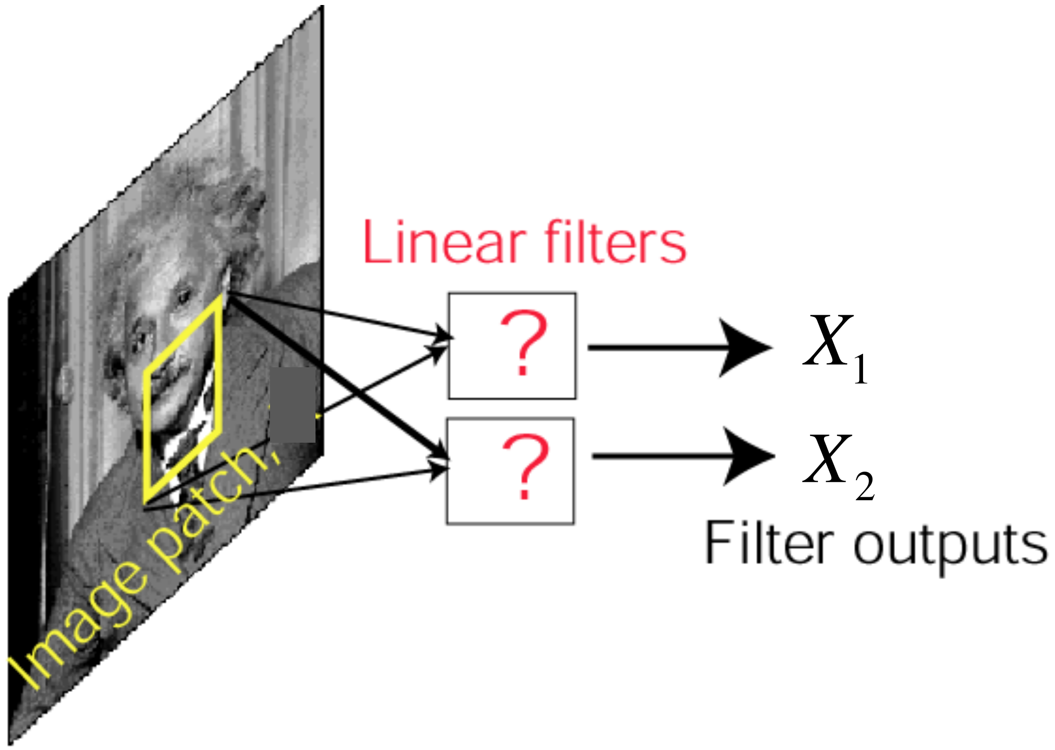
Bottom-up approach

Choose and manipulate projections, to optimize probabilistic and information-theoretic metrics



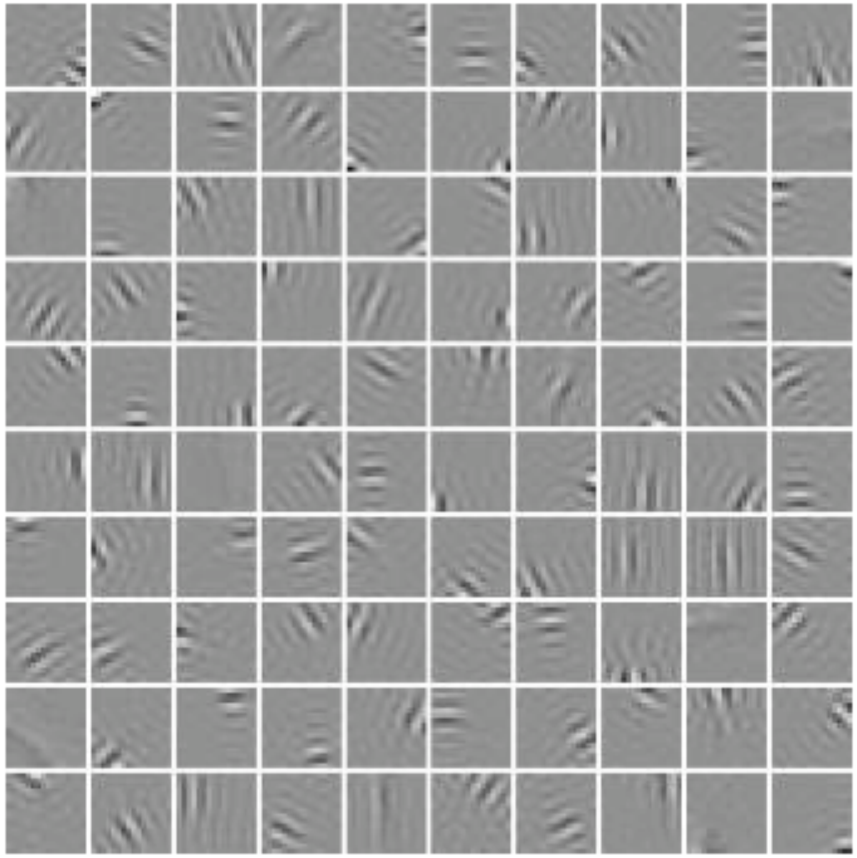
Assuming a **linear** system and optimizing joint statistics: **independence**

Linear Model: Theory



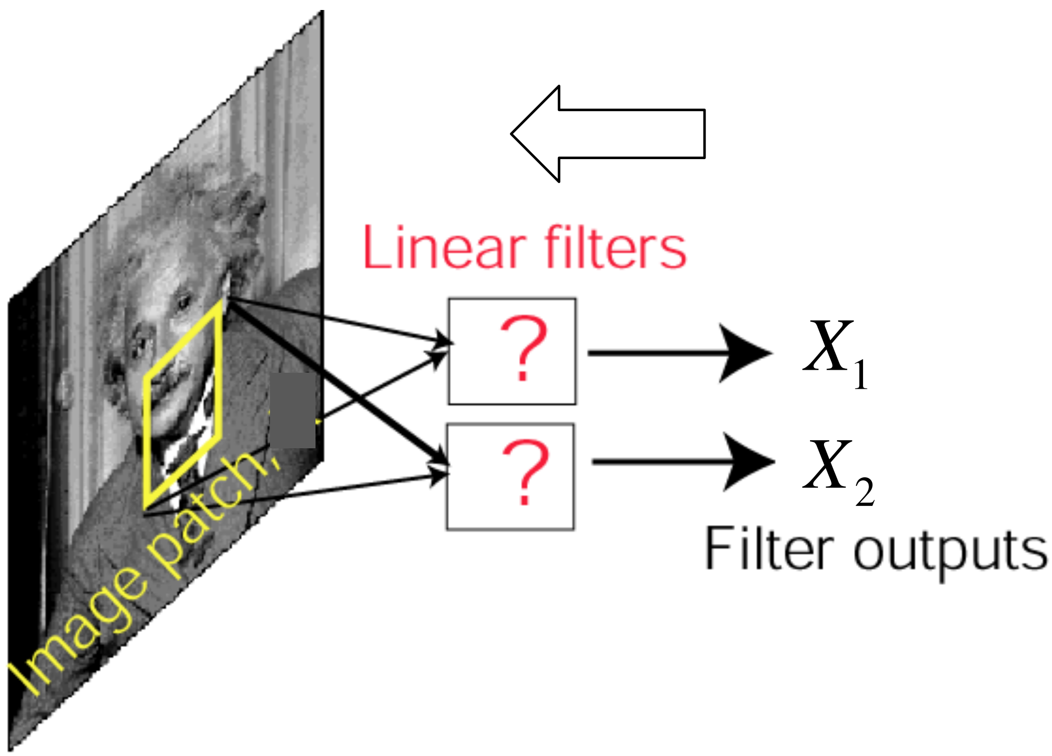
Find linear filters that maximize measure of statistical independence (or sparseness) between filter outputs to natural images (e.g., *Olshausen* & Field, 1996; *Bell* & Sejnowski 1997)

Linear Model: Theory



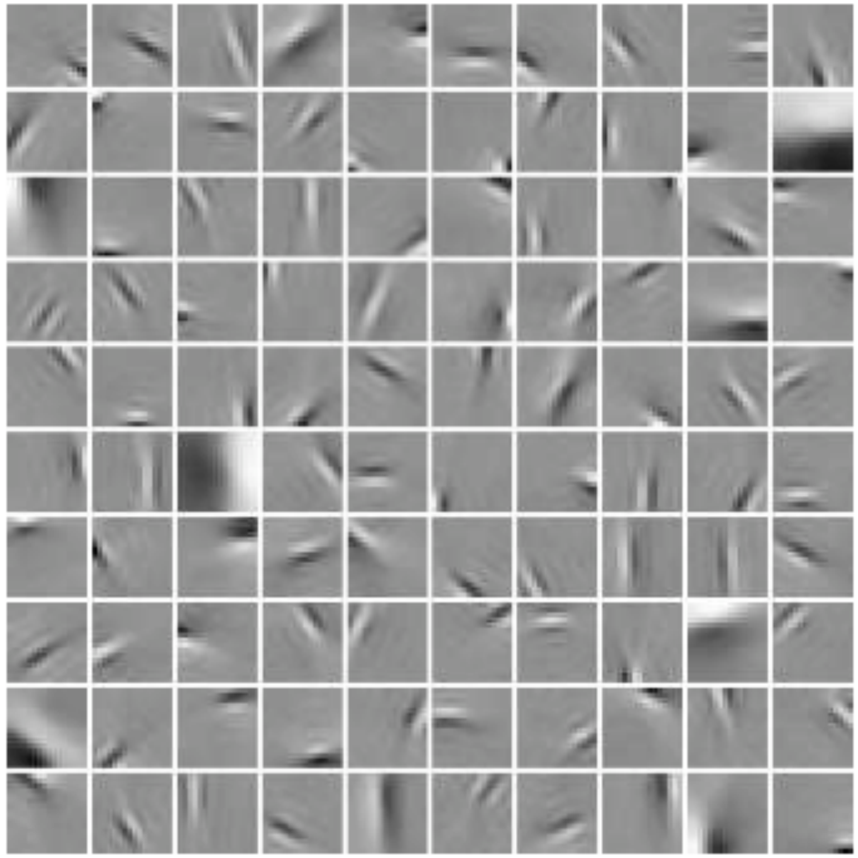
- ICA filters plotted from Hoyer images (e.g., *Olshausen* & Field, 1996; *Bell* & Sejnowski 1997— *here at Redwood*!)
- Qualitatively related to V1 RFs

Linear Model: Theory



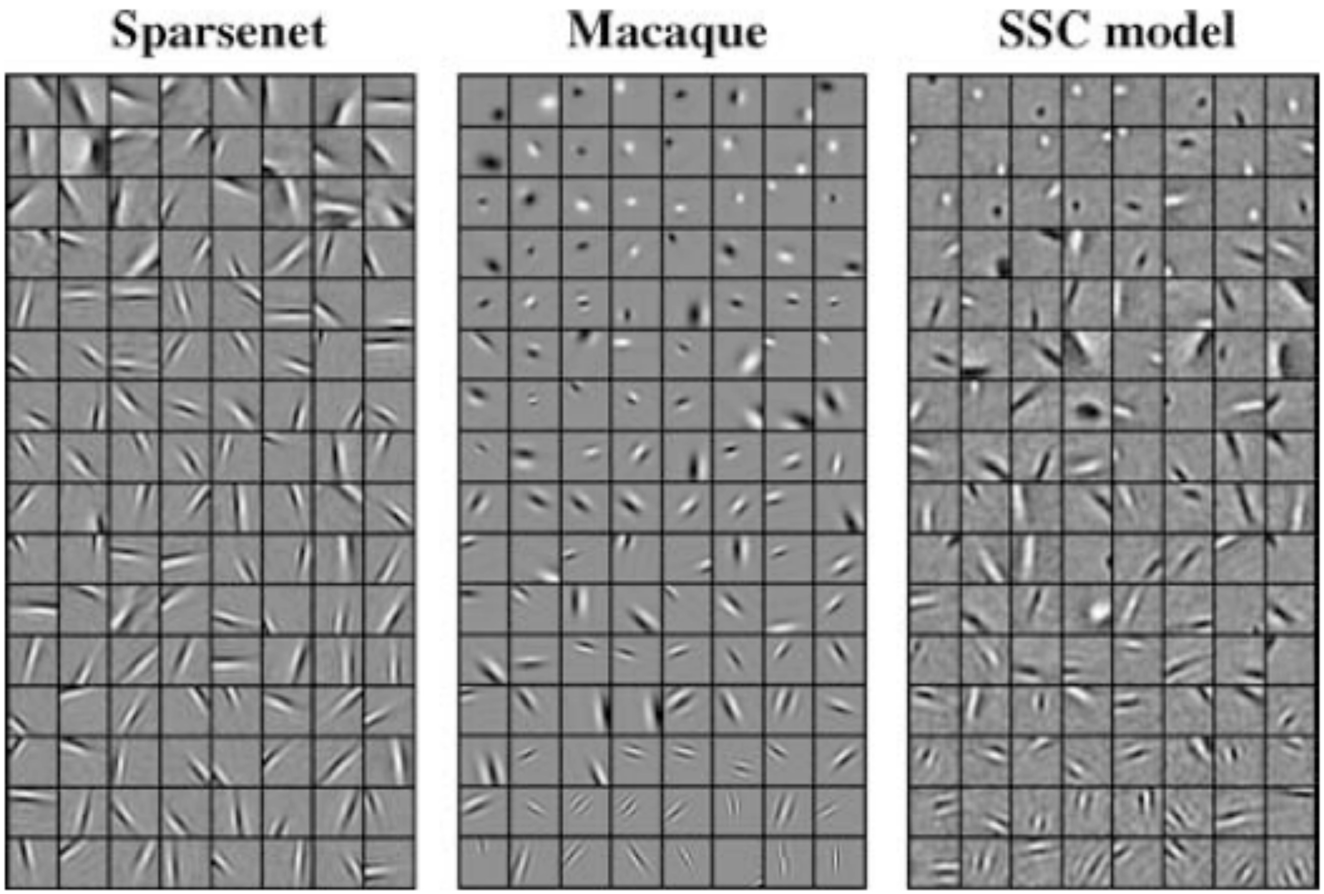
Linear transform, so from filter outputs can also go back to the image…

Linear Model: Theory



- ICA basis functions; from Hoyer
- Olshausen & Field, 1996; Bell & Sejnowski 1997

Linear Model: Theory

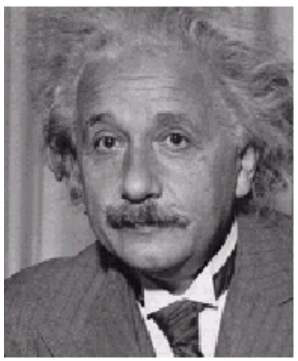


- Note also more recent work explaining neural diversity
- Rehn and Sommer, 2007 (data: Ringach)

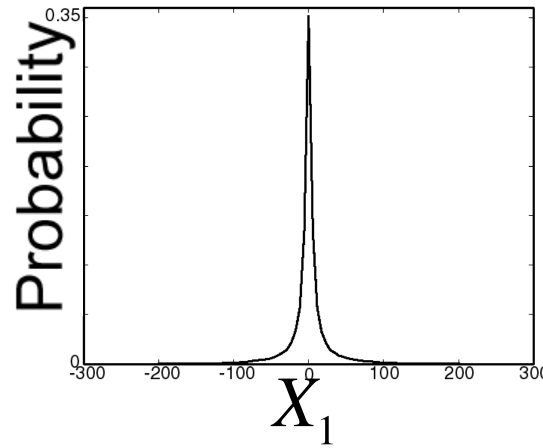
Bottom-up approach

What about sparse?
(e.g., Olshausen & Field)

Bottom-up Statistics

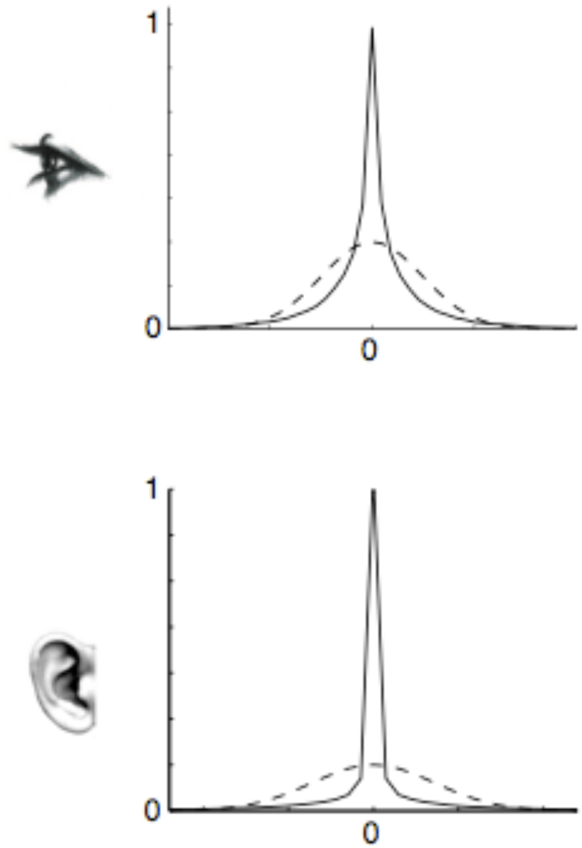


X_1



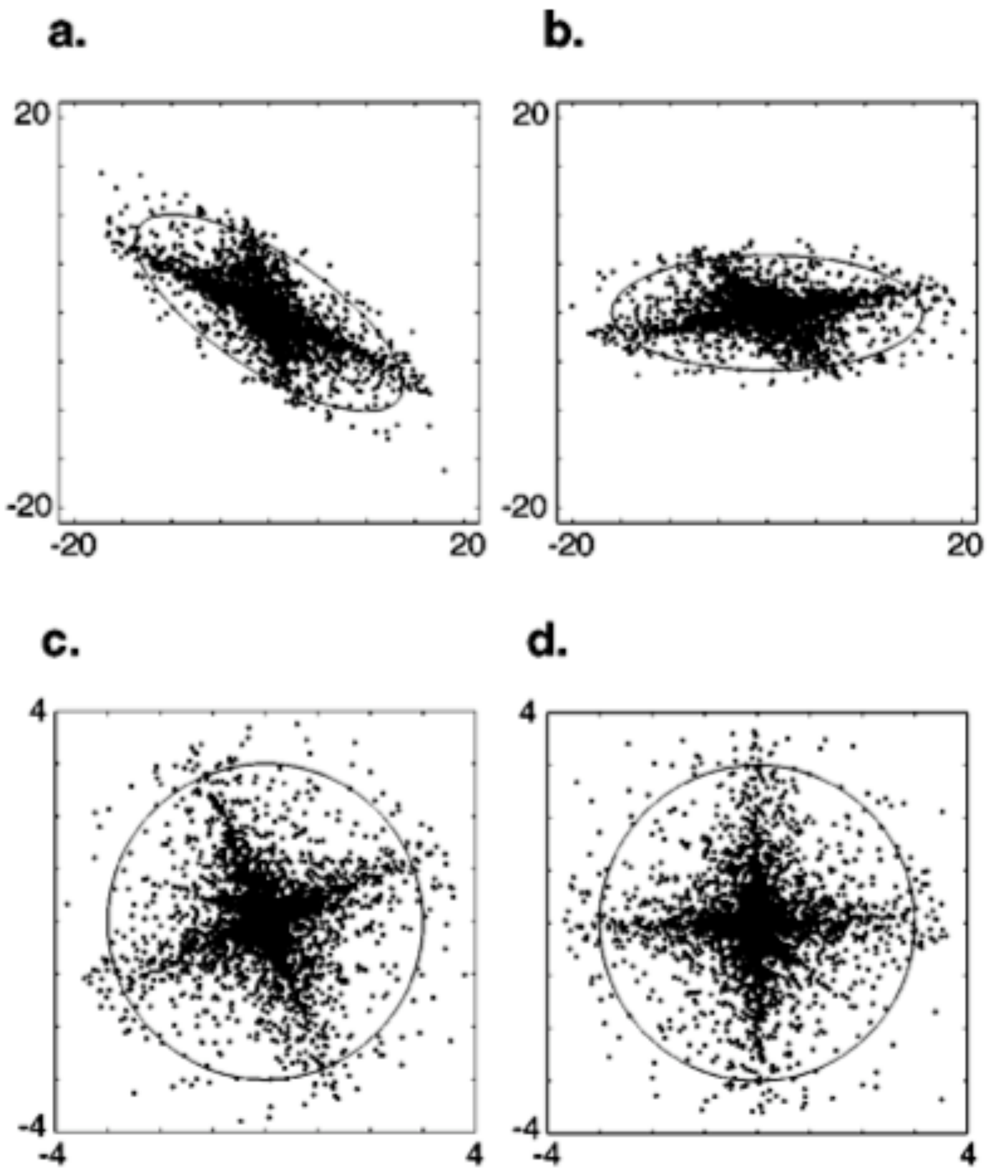
After Field, 1987

Bottom-up Statistics



- Well described by, eg, generalized Gaussian distribution

Geometric view (ICA)



Non Gaussian
(sparse)
distribution

Simoncelli & Olshausen
review, 2001

ICA and sparse coding

- In ICA maximizing independence assuming a linear transform (e.g., by maximizing joint entropy of the output).

- But should also assume that the outputs have a sparse distribution…

Summary

- Different levels of modeling…
- We've considered bottom-up scene statistics, efficient coding, and relation of linear transforms to visual filters
- Efficient coding through one channel and multiple channels
- Can we propagate statistical principles (such as efficient coding?) and how far?
- Next class: nonlinearities