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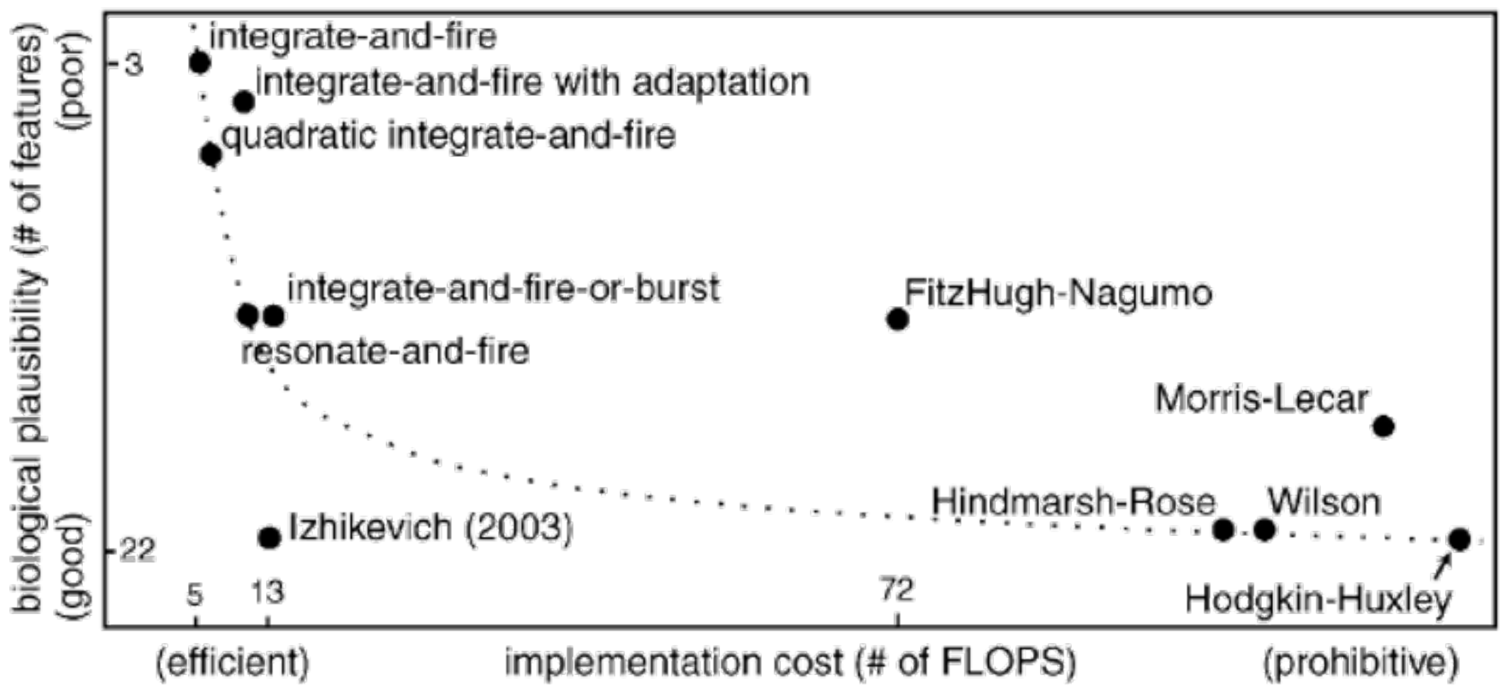
- Simple (Still widely used today in brain modeling, scales up to networks of neurons)

Integrate and Fire



https://encyclopedia.pub/147

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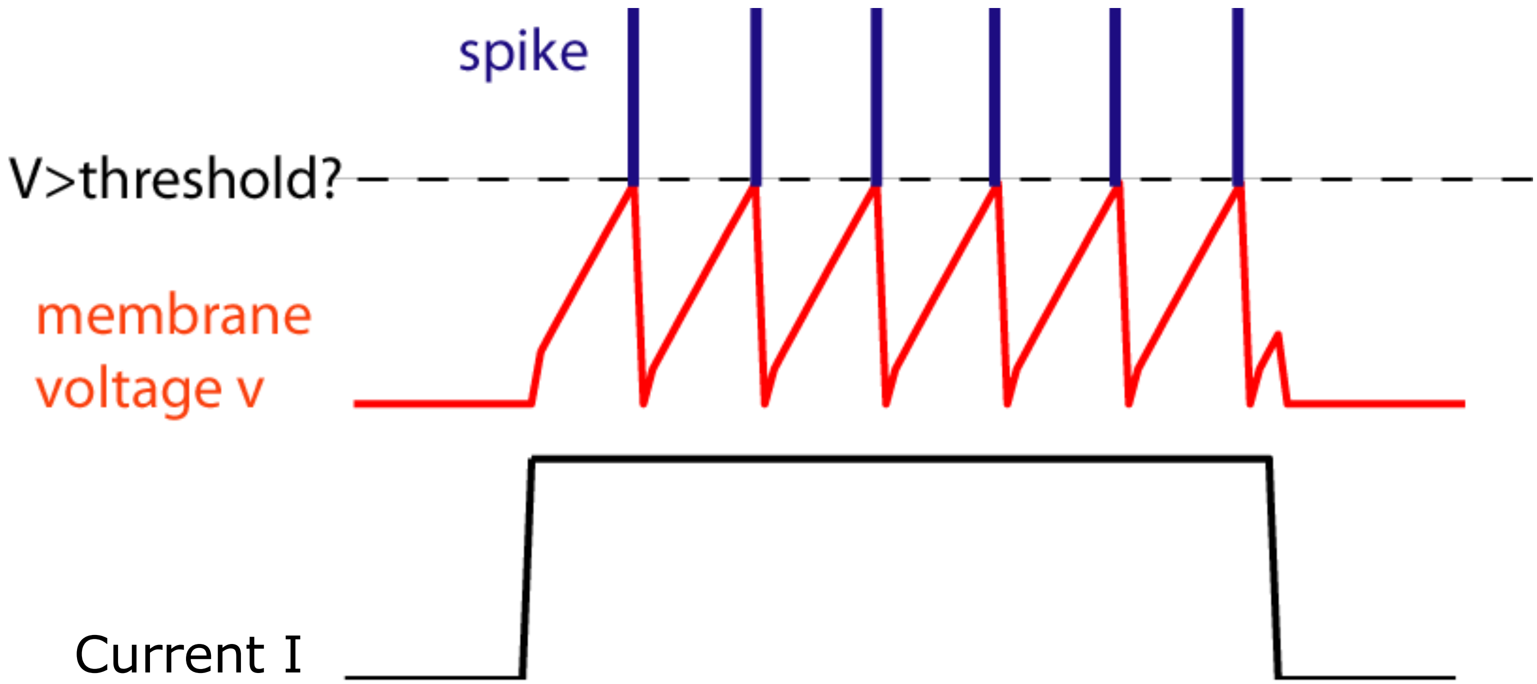
- Parameters correspond to known properties of neurons (and electrical circuits)
- Simple (doesn't model biophysical detail)
- Simple (DE can be solved, eg, using separable DE!)
- Simple (Still widely used today in brain modeling, scales up to networks of neurons)
- Was used, for instance, in Eliasmith paper we went through

Membrane voltage and spiking



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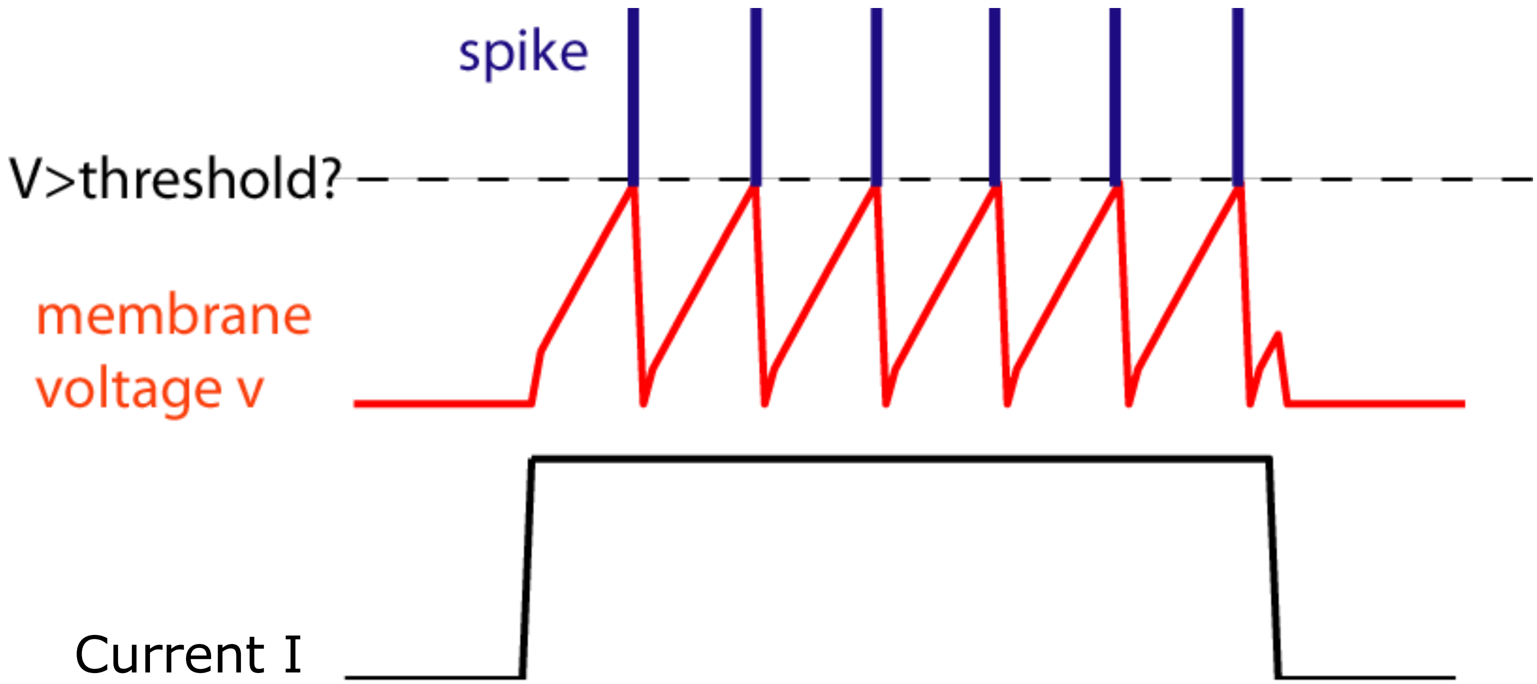
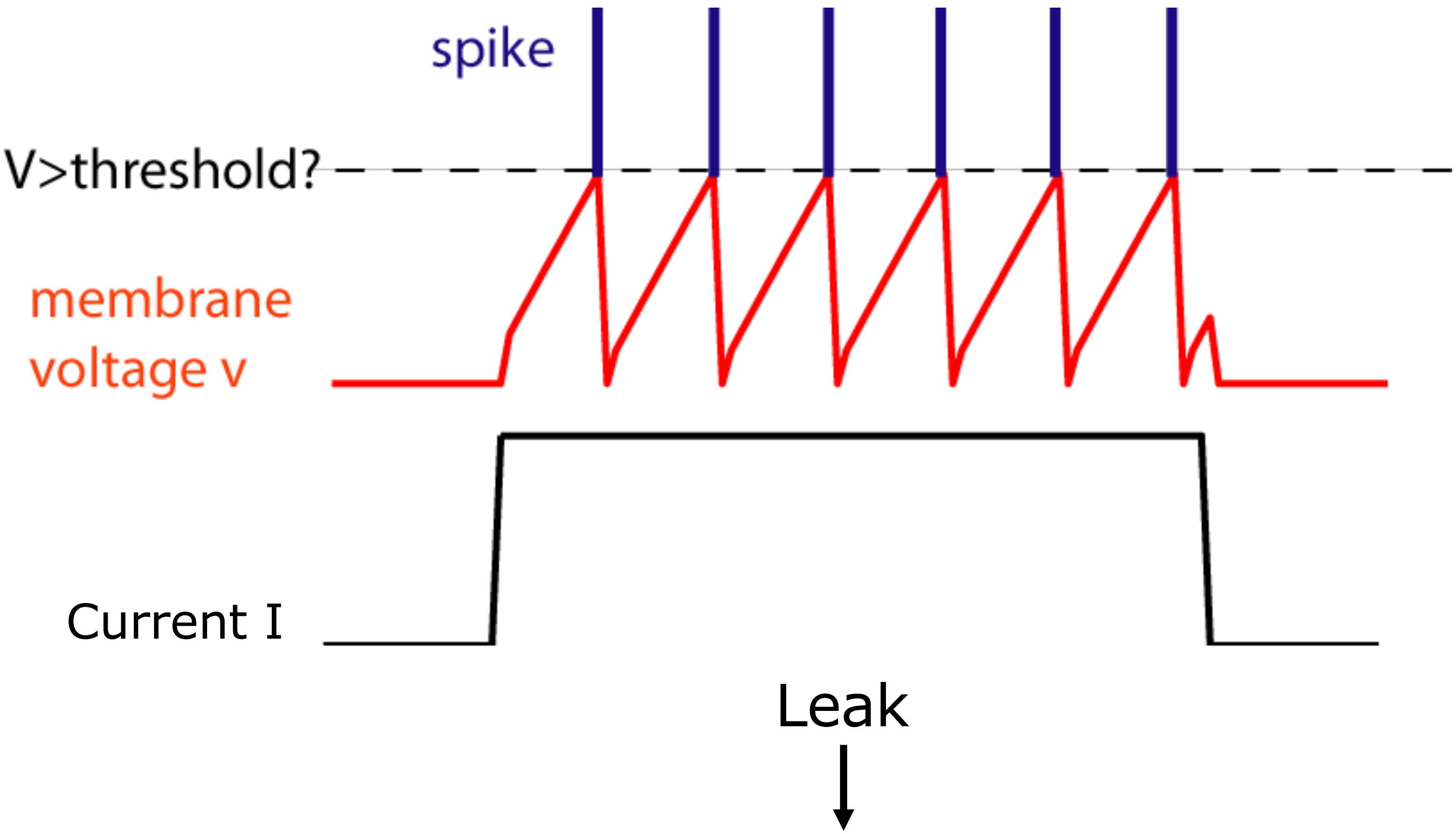


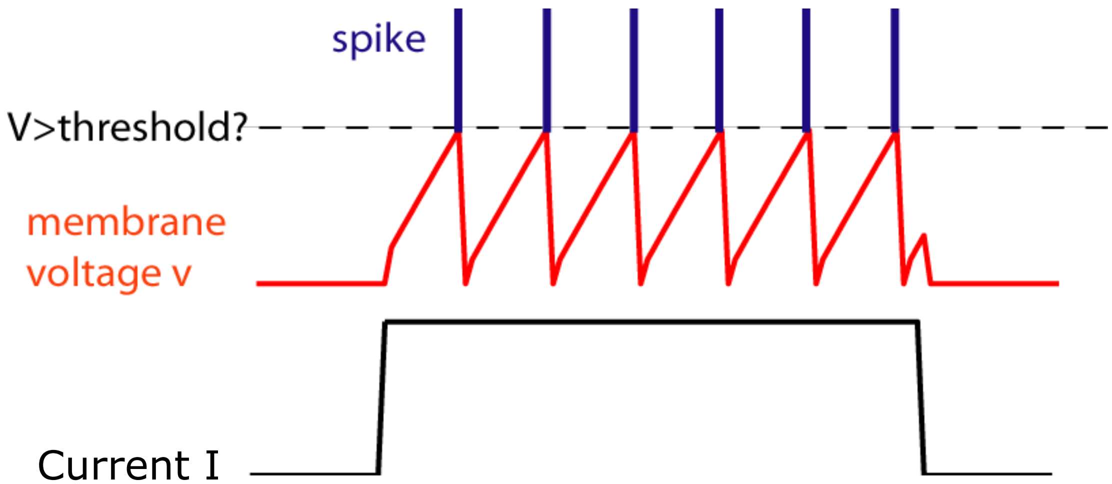
Input current increases the membrane potential

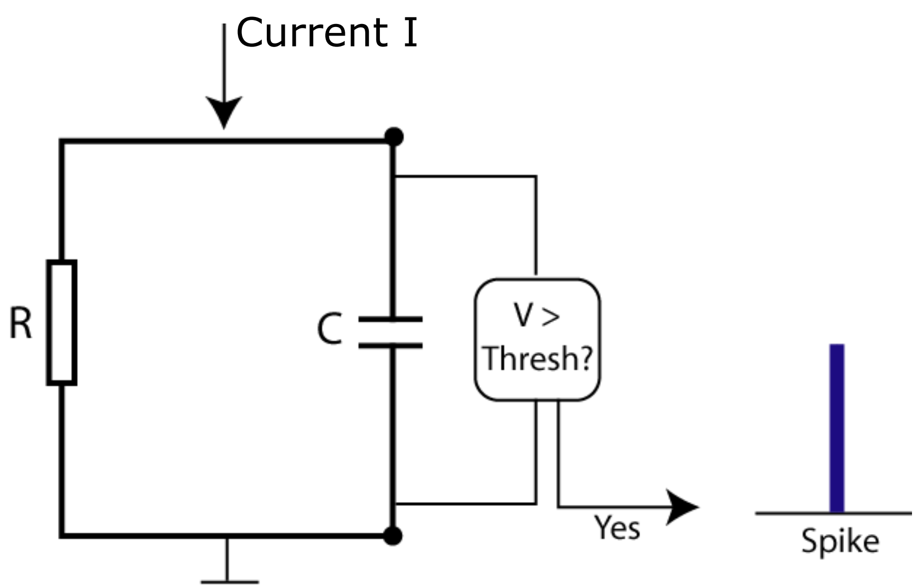
Leaky Integrate and Fire



$$\frac{dv}{dt} = \frac{-v}{\tau} + \frac{I}{C}$$

Membrane voltage and spiking





If above spike threshold, generate a spike and reset the membrane potential

Separable DE's

- Definition: $f(y)dy = g(x)dx$ (example: $\frac{dy}{2y^2} = xdx$

(a) Get equation in separable form (y's on the left; x's on the right)

(b) Integrate both sides (don't forget constant of integration c)

$$\int f(y)dy = \int g(x)dx + c$$

(c) Plug in initial condition, (example: y(0)=5), and find constant of integration c.

(d) Solve for y, by plugging constant c into result of (b)

Leaky Integrate and Fire DE

- DE $$\frac{dv}{dt} = \frac{-v}{\tau} + \frac{I}{C}$$
- Change with time: v(t), t
- Assume constants: I, R, C, $\tau = RC$
- Putting in separable form and solving

$$v(t) = v(t=0)e^{-t/\tau} + RI(1 - e^{-t/\tau})$$

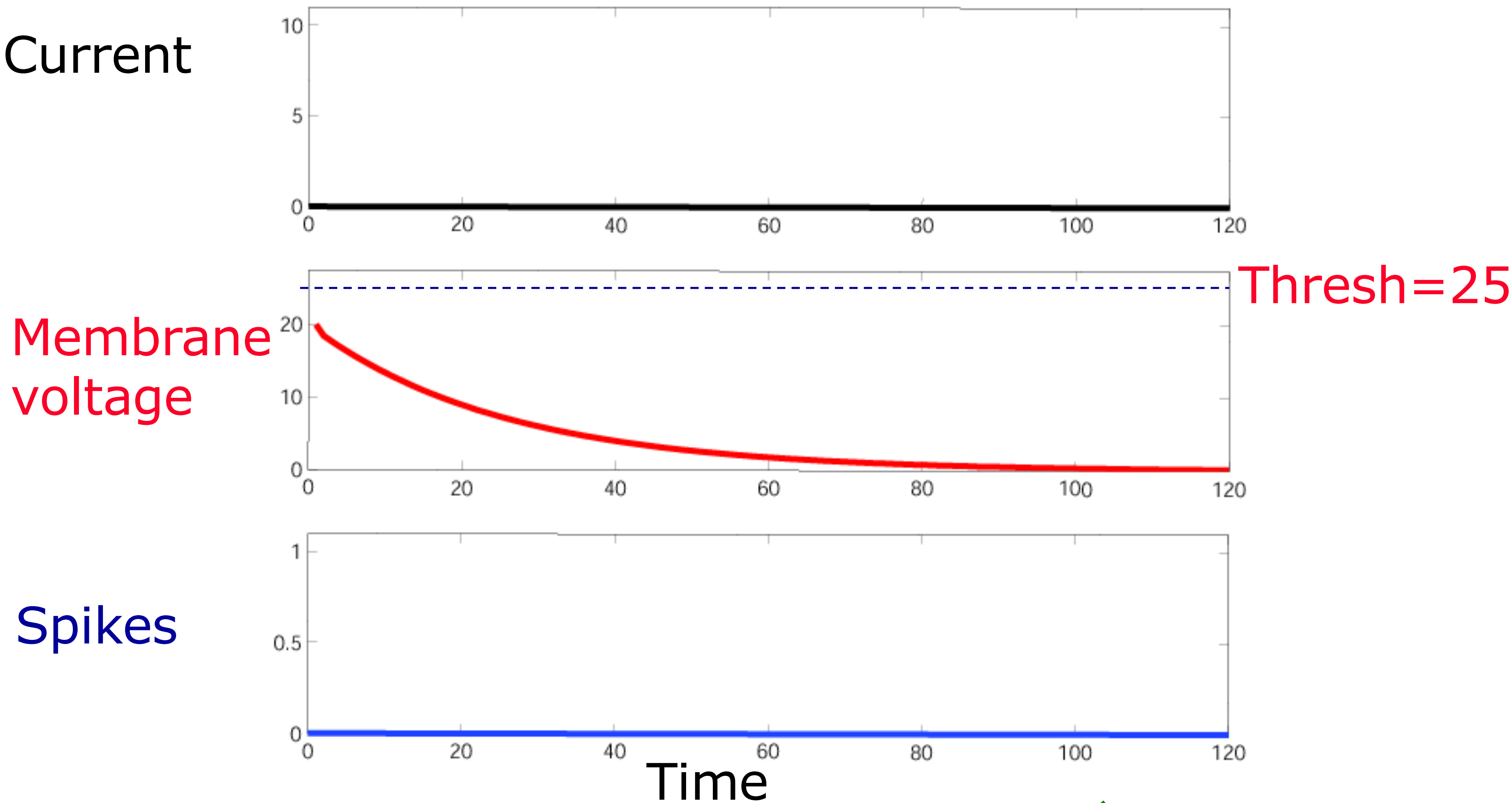
Leaky Integrate and Fire Circuit

- Solution to DE

$$v(t) = v(t=0)e^{-t/\tau} + RI(1 - e^{-t/\tau})$$

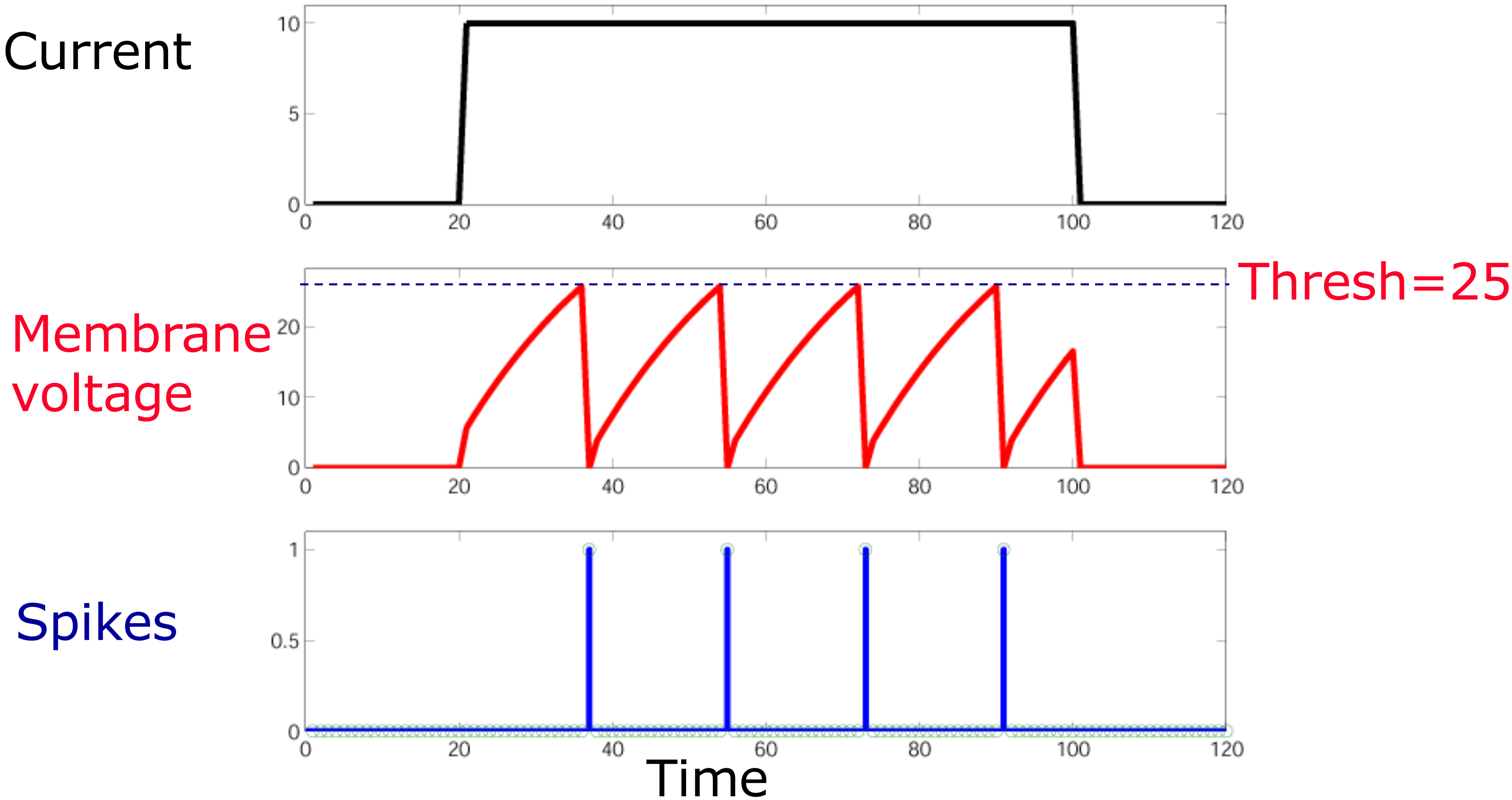
- After action potential, v reset to v(t=0), and time reset to 0.

NO CURRENT I



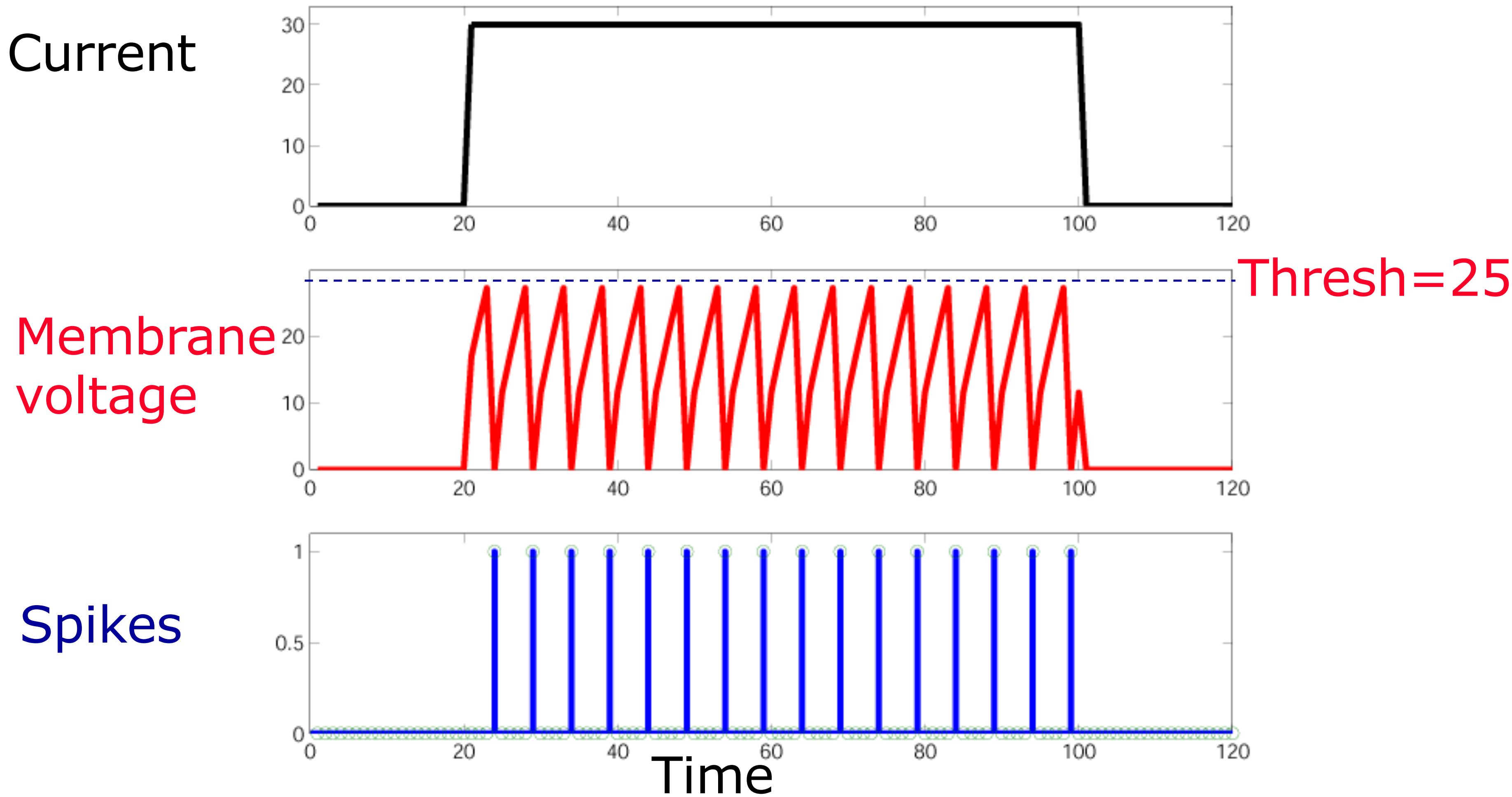
$$v(t) = v(t=0)e^{-t/\tau} + RI(1 - e^{-t/\tau})$$

WITH CURRENT I and V(t=0)=0



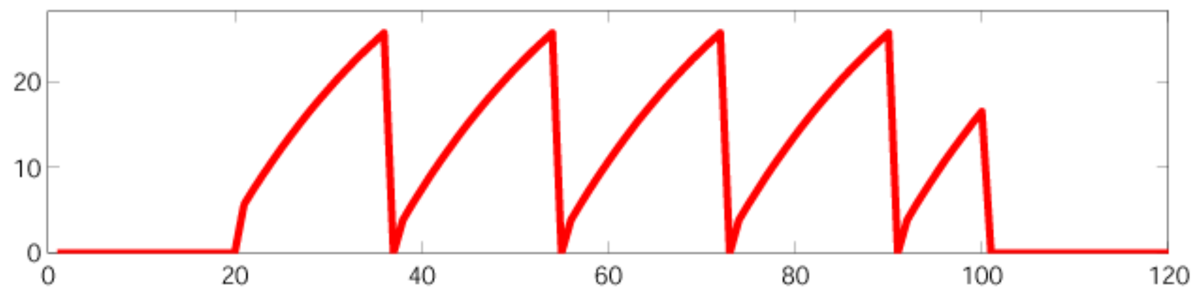
$$v(t) = v(t=0)e^{-t/\tau} + RI(1 - e^{-t/\tau})$$

INCREASE CURRENT I



$$v(t) = v(t=0)e^{-t/\tau} + RI(1 - e^{-t/\tau})$$

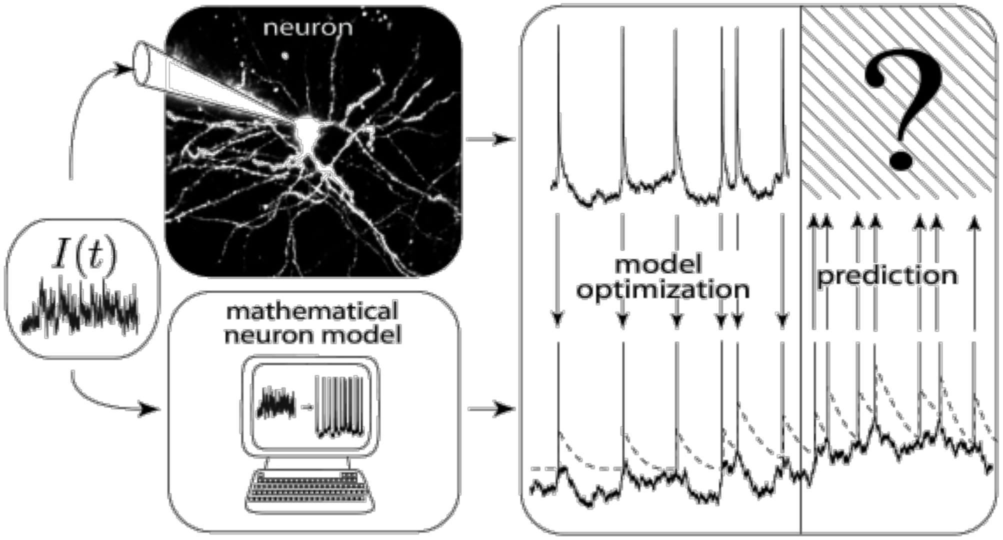
Membrane potential



Note: For simplicity, we started membrane potential at 0, so Y axis starts from 0. Note that we could change this, and that the usual resting membrane Potential is negative millivolts (e.g., -70)

https://faculty.washington.edu/chudler/ap.html

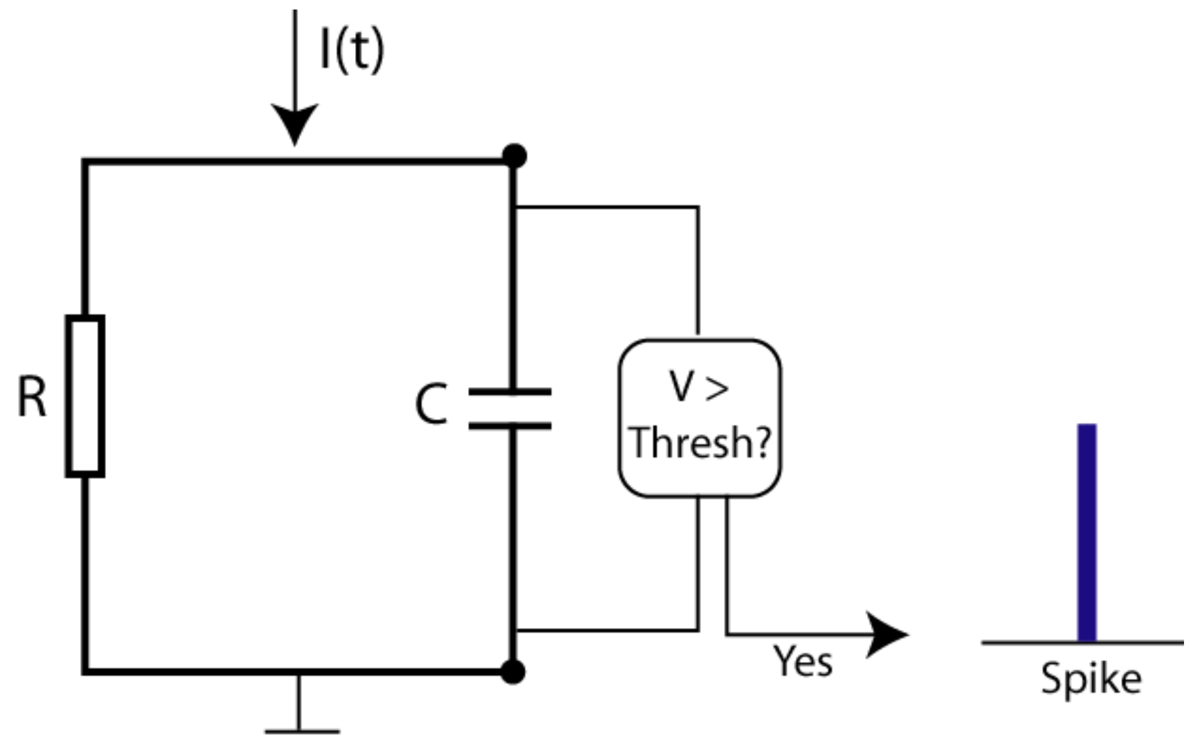
Integrate and Fire



Note: Read some more about Integrate and Fire:

https://neuronaldynamics.epfl.ch/online/Ch1.S5.html

Leaky Integrate and Fire Circuit

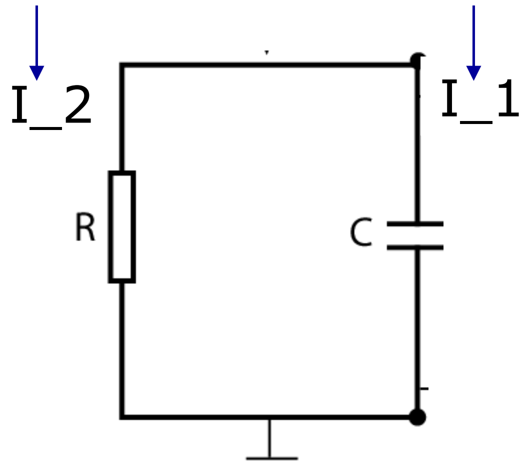


Leak

Current

$$C\frac{dv}{dt} = \frac{-v}{R} + I(t)$$

RC Circuit

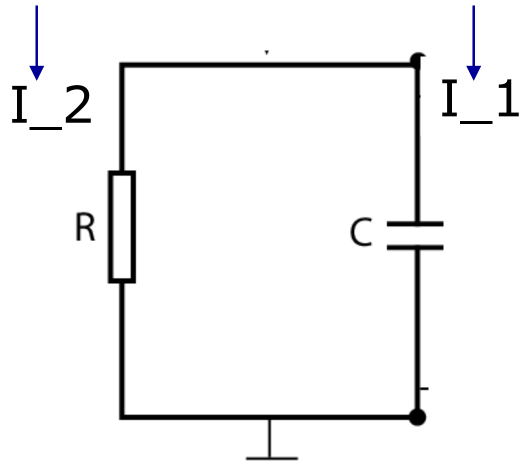


(1) C V = Q (C capacitance; V voltage; Q charge)

(2) I_1 = dQ/dt (I_1 current)

(3) C dv/dt = I_1 (taking derivative in (1) and plugging in (2))

RC Circuit



(4) V = I_2 R (Ohms law)

(5) I_2 = V/R (I current)

(6) I_1 + I_2 = 0 (Kirchkoff's law)

(7) -V/R = C dV/dt (plugging (3) and (6))

(8) Define $\tau = RC$ (time constant!)

Time constant

$$\tau = RC$$

(1) Q=CV

(2) dQ/dt=I
Q=It + const

(3) C=Q/V = It/V

(4) R=V/I

(5) RC = (V/I)(It/V) = t (time units!)

[V volt; C Farad; R Ohm; I amper; Q Coulomb]