$\boxed{\text{Burt Rosenberg}}$

Problem Set 1

OUT: 8 SEPTEMBER, 1997
DUE: 17 SEPTEMBER, 1997

Announcements

Class on Wednesday Sept. 9, Friday Sept. 11 and Monday Sept. 15 are canceled.

Assignment

1. Give formulas using $\wedge, \vee$ and $\neg$ for all 16 of the Boolean functions on two variables. Use the notation F_{abcd} where each of a, b, c and d are either T or F, and:

$$\begin{aligned} F_{abcd}(F,F) &= a \\ F_{abcd}(F,T) &= b \\ F_{abcd}(T,F) &= c \\ F_{abcd}(T,T) &= d \end{aligned}$$

   Examples: $F_{FFFF}(A,B) = \mathrm{F}$, $F_{TFFF}(A,B) = \neg(A \vee B)$.

2. Prove that DeMorgan's Theorem holds for 3-way conjunctions and disjunctions:

$$\begin{aligned} \neg(A \wedge B \wedge C) &= (\neg A) \vee (\neg B) \vee (\neg C) \\ \neg(A \vee B \vee C) &= (\neg A) \wedge (\neg B) \wedge (\neg C) \end{aligned}$$

3. Give a formula using $\wedge, \vee$ and $\neg$ for the *Majority function* on Boolean inputs A, B and C and output D defined as:

$$\mathrm{Maj}(A,B,C) = \begin{cases} \mathrm{T} & \text{Two or more of } A,\ B \text{ and } C \text{ are T} \\ \mathrm{F} & \text{otherwise} \end{cases}$$

4. Give a formula using $\wedge, \vee$ and $\neg$ for the *Maximum function* on Boolean inputs A_1, A_2, A_3 and A_4 and outputs B_1, B_2, B_3 and B_4, defined as:

$$\mathrm{Max}(A_1,A_2,A_3,A_4) = \begin{cases} B_i = \mathrm{T} & \text{If } A_k \text{ is True, for some } k, \text{ and } i = \max_k\{\, A_k = \mathrm{T} \,\} \\ B_j = \mathrm{F} & \text{otherwise} \end{cases}$$

5. Draw the 4-bit adder, and label the wires with T or F to demonstrate the addtions:
   (a) 1+1
   (b) 2+3
   (c) 5+6
   (d) 9+7